

Fracture Mechanics Life Analytical Methods Verification Testing

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INTRODUCTION

NASCRAC™ (NASA Crack Analysis Code - Version 2.0) is a second generation fracture analysis code developed for NASA/MSFC. The code uses a weight function approach to solve traditional fracture problems such as stress intensity factors and life calculation due to fatigue. NASCRAC also contains capabilities for advanced fracture analysis, e.g., crack retardation, life calculation due to creep, and elastic-plastic stress redistribution near the crack tip. Since NASCRAC includes the computationally efficient weight function approach and a broad spectrum of advanced capabilities, NASA/MSFC expects to employ NASCRAC as an integral component of the NASA Fracture Control Program for validating flight hardware. This critical role of NASCRAC in future NASA analyses dictates both a complete and objective independent verification and validation (V/V) of the code to ascertain the restrictions and ranges of applicability for each NASCRAC solution. Nichols Research Corporation (NRC) and its subcontractor, Cornell University, and consultant, Fracture Analysis Consultants (FAC), have been contracted by NASA/MSFC to perform such a V/V. This report discusses methodology and interim results from the V/V effort.

The current effort focuses on verification and validation of solutions embedded in NASCRAC. No attempts are made to correct solutions or to develop new solutions. In the case of minor programming errors, corrected versions are run offline to determine the extent of the problem.

The level of effort has been divided into two distinct areas: 1) literature research/analysis, and 2) testing. NRC has been the lead in the first effort with assistance from FAC. Cornell has performed the testing effort.

The NASCRAC V/V plan partitioned the available solutions into three groups: *basic information (BI)*, *synthesized results (SR)*, and *advanced capabilities (AC)*. This report will only discuss results from the BI group, including K vs a , J vs a , and *crack opening area (COA) vs a* . Section II provides a succinct description of the theory behind NASCRAC. Section III focuses on the V/V methods and decision process being used by NRC. Interim results are presented in Section IV. Recommendations and a mid contract summary are included in Section V. Throughout the report the letter K implies K_I , the opening mode stress intensity factor.

TECHNICAL BACKGROUND

NASCRAC is a fracture analysis code capable of performing linear elastic and elastic-plastic fracture analyses. NASCRAC is restricted to mode I, or opening mode, fracture.

Capabilities incorporated into NASCRAC include computation of K vs a , J vs a , COA vs a , *fatigue crack growth*, *tolerable crack size*, *creep crack growth*, *proof test logic*, *tearing instability*, and *localized elastic-plastic stress redistribution*. NASCRAC can accept cyclical, steady-state, and random load spectrum definitions. Eleven material libraries are available: two miscellaneous steel libraries, stainless steel, AL-2024, AL-6061, AL-7075, two miscellaneous aluminum libraries, cast aluminum, inconel, and titanium. Users may also define a material interactively or create a material library. Currently twenty-eight crack configurations are incorporated in the code. Crack retardation is possible using either the Wheeler or Willenborg models.

K solutions in NASCRAC are computed using encoded closed form solutions for uniform tensile loads and weight function formulations for arbitrary loads. Robust integration routines incorporating Gaussian integration and a broad library of weight functions provide an extensive computational capability for calculating K solutions of various loadings and geometries. In the weight function approach, a K solution of a specific geometry can be calculated for an arbitrary loading by integrating a point load solution over the crack face. This approach can be expressed as:

$$K = \int_0^a \sigma(x) h(x,a) dx$$

where a = crack length
 $\sigma(x)$ = crack plane stress derived from the uncracked geometry
 $h(x,a)$ = weight function from a known solution

Weight functions can be determined from simple load cases and applied to unique, complex load cases. Figures 2-1 and 2-2 illustrate the weight function approach to fracture analysis. As shown in Figure 2-1, K solutions can be obtained for an arbitrary loading by employing superposition to reduce the arbitrary loading to two simpler loadings: a cracked geometry with external tractions (the problem of interest) and an identical cracked geometry with tractions only along the crack face. Since these two loadings are reduced from an uncracked problem, their K solutions sum to zero, i.e., $K_d = -K_e$.

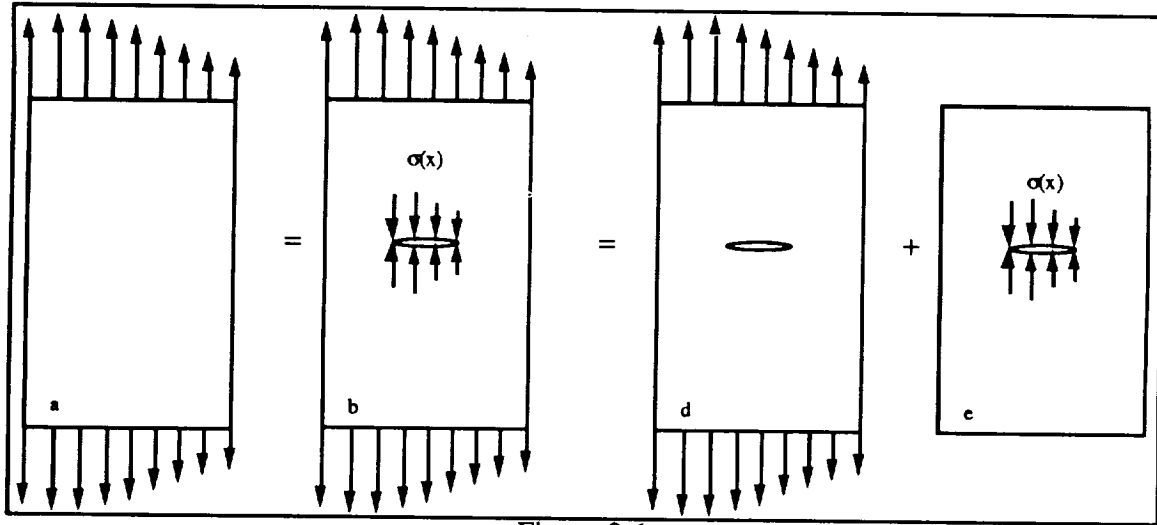


Figure 2-1
Application of Superposition Principle in Fracture Mechanics

As depicted in Figure 2-2, $-K_e$ can be calculated from a weight function formulation. The weight function solution is calculated by integrating the product of the crack face stress distribution $\sigma(x)$ and the weight function $h(x,a)$ along the crack face.

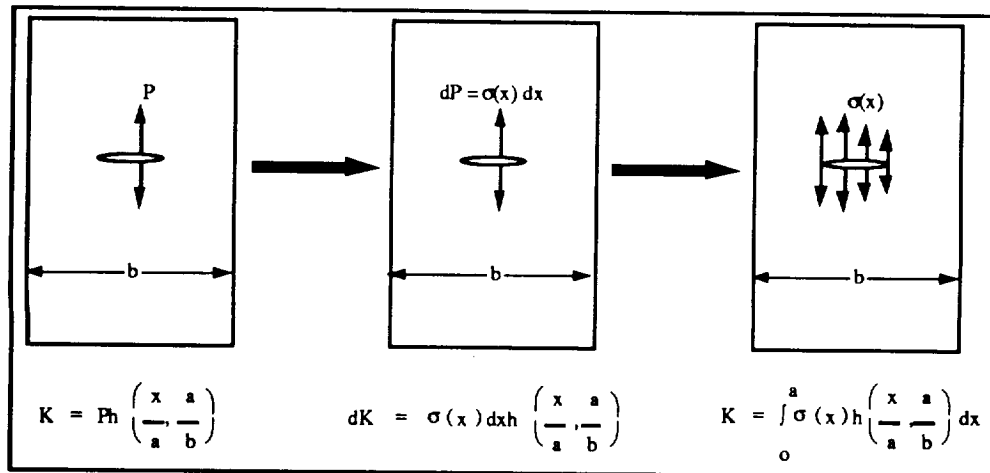


Figure 2-2
Weight Function Formulation for Stress Intensity Solutions

J-integral solutions in NASCRAC are computed by assuming J to be a summation of elastic and fully plastic components:

$$J = J_e + J_p$$

where

J = the total J integral

J_e = the elastic component of the J integral

J_p = the plastic component of the J integral

The elastic component is computed by using an effective crack length with a standard K solution and the plastic component is computed from a limit load concept using a calibration factor obtained from handbook solutions. A Ramberg-Osgood constitutive relationship is used to define plasticity. J_p values generally require interpolation because the handbook solutions are limited in range. The general equation for J_p is given as:

$$J_p = \alpha \sigma_y \epsilon_y c \frac{a}{b} h_1 \left(\frac{P}{P_0} \right)^{n+1}$$

In this equation α is a material property; σ_y and ϵ_y are the yield stress and strain of the material; a , b , and c are geometric dimensions with a being the crack length; P and P_0 are the applied load and limit load of the structure, h_1 is a correction factor related to geometry and strain hardening of the material, and n is the strain hardening exponent from the Ramberg-Osgood model.

In NASCRAC, five configurations have an option for calculating crack opening area. For each configuration, the crack opening area is calculated according to closed form solutions listed in reference 10.

Seven of NASCRAC's configurations include a variable thickness option for calculating a K solution and life due to fatigue crack growth. The option is a discrete variable thickness with the thickness being defined at specified points along the crack plane. During calculation of K, the stress is distributed along the crack in proportion to the thickness at the discrete points.

Three types of load spectrums can be input into NASCRAC: cyclical, steady-state, and random. For the cyclical spectrum, load transients are defined with a specified number of cycles. Transients are arranged into blocks to form the spectrum. To define a load, the user must input two of the following five variables: maximum stress, minimum stress, stress range, stress mean, and R ratio.

VERIFICATION AND VALIDATION METHODOLOGY

The NASCRAC V/V plan is a comparative approach using three different types of reference solutions: 1) documented solutions from the literature, including closed form and graphical solutions, 2) finite element and boundary element solutions, and 3) testing. NASCRAC solutions were categorized into three areas: *Basic Information (BI)*, *Synthesized Results (SR)*, and *Advanced Capabilities (AC)*. The *BI* category includes K vs a , J vs a , and crack opening area (COA) vs a . The *SR* category includes life calculations due to fatigue and creep, tolerable crack size, proof test logic, and tearing instability. The *AC* category includes elastic-plastic stress redistribution and crack transition. This report presents methodology and results solely from the *BI* category.

A minimum of 422 solutions are available in NASCRAC. This quantity was derived by summing the number of crack topologies available for each NASCRAC capability. Variations in loading conditions were not included in the tabulation. Each NASCRAC solution group, i.e., *BI*, *SR*, and *AC*, requires a different V/V approach. *BI* solutions are dependent on analytical, numerical, and experimental results external to NASCRAC plus the weight function feature of NASCRAC. Solutions in the *SR* category use a number of programmed theoretical or empirical crack-growth rate and stability models (e.g., Paris's equation) plus data calculated or interpolated from *BI* results to synthesize or compute results. An accurate *SR* depends on the accuracy of the *BI* and also on the proper choice of a theoretical or empirical model for the physical problem. Thus, verification of *BI* solutions can be accomplished with literature and numerical analyses whereas verification of an *SR* solution requires verifying the *BI* and determining the applicability of the chosen empirical or theoretical model using experimental and numerical techniques. *AC* solutions (spectrum loading, elastic-plastic stress redistribution, crack transition) require *BI* results and advanced theoretical formulations. Accurate *AC* solutions are strongly dependent on understanding the range for which the formulation is applicable.

A three-step V/V procedure has been used for the *BI* solutions: 1) check coded equations, 2) check weight function capability, and 3) check variable thickness functionality. The coded equations have been checked by review of the NASCRAC source code or by comparison of NASCRAC results to literature sources. If reasonable agreement between NASCRAC and the literature source is not found, an independent solution using finite element analysis has been generated. Agreement between the literature source and the finite element solution indicates an error in NASCRAC. Agreement between NASCRAC and the finite element solution suggests a problem with the literature source. Such disagreement with the literature has been resolved by obtaining a second literature source.

Independent integration external to NASCRAC was used to check the NASCRAC integration routines. The external routines were based on a Romberg integration algorithm which differed from the Gaussian algorithm in NASCRAC.

The accuracy of NASCRAC's ability to estimate K solutions for variable thickness planar bodies using weight function solutions has been determined by comparing NASCRAC results with finite element results. The finite element models included up to third order polynomial variation in global thickness.

Several references have been used extensively for V/V of the *BI* solutions. For K vs a solutions and uniform or bending loads, references 10 and 12 provided graphical, curve fit, and closed form solutions. Reference 12 also contained closed form point load solutions for certain NASCRAC configurations. These point load solutions were integrated numerically to verify the NASCRAC weight function solution. Reference 14 was also a primary reference for weight function solution V/V. For several of the non-through crack

K vs a , J vs a , and crack opening area (COA) vs a solutions, references 6 and 11 were critical resources.

References 5 and 6 were the primary V/V sources for NASCRAC's seven J vs a configurations. These two references listed the coefficient tables coded into NASCRAC in addition to the coded equations.

Three different NASCRAC BI sections were verified by direct comparison of coded equations with literature sources. In the 100 series configurations (ASTM standard fracture toughness specimens), the coded equations were compared to reference 2, ASTM E 399. For the J-integral capabilities, the coded limit load (P_0) equations were compared to reference 5. Finally, for the five COA vs a configurations, the coded equations were compared to equations listed in references 12 and 14.

FRANC and FRANC-3D, fracture specific finite element codes, were employed in the K vs a V/V efforts. These workstation based codes allow an analyst to compute stress intensity factors for arbitrary cracks in arbitrary bodies. Menu-driven post-processing tools provide both numerical and graphical results.

RESULTS AND DISCUSSION

The techniques described in the previous section have been used to verify and validate the K vs a , J vs a , and COA vs a solutions in NASCRAC. Results have ranged from identical and acceptable solutions versus references to coding errors, documentation errors, and unacceptable solutions. This section presents V/V results of NASCRAC solutions in which errors and/or recommendations have been determined.

K vs a -- UNIFORM THICKNESS

The uniformly thick K solutions in NASCRAC are the key to the code's capabilities. There are twenty-eight (28) uniformly thick K solutions. These solutions permit static checks of K versus K_{Ic} and also drive the fatigue crack growth and the tolerable crack size capabilities.

100 Series Results

Configuration 101 (Compact Tension Specimen)

The geometry for configuration 101, Compact Tension Specimen, is shown in Figure 4-1 below. A minor error was detected in configuration 101. The error is a typographical error in the first coefficient of the FAOW equation. The coefficient should be 0.886 but the NASCRAC value is 0.866 (Figure 4-2).

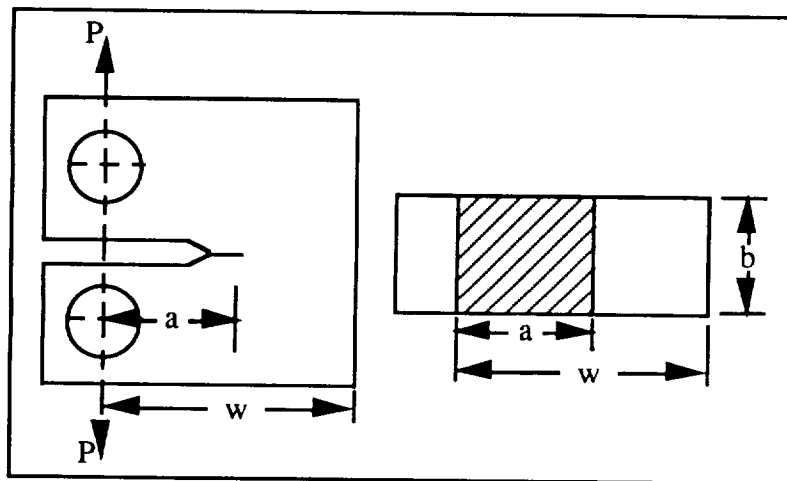


Figure 4-1
Geometry for NASCRAC Configuration 101

```

SUBROUTINE K100
C-----C
C-----C
  AOW=ANOW(1)/WIDTHS(1)
  SIGZ=EQPARS(ITRANS,IDEF,1)
C
  GOTO (101,102,103,104) (KRKTYP-100)
C
C  COMPACT TENSION SPECIMEN , KRKTYP=101
C
101 FAOW= 0.866 +AOW*(4.64 +AOW*(-13.32 +AOW*(14.72 +AOW*(-5.6))))
  FAOW=FAOW*(2.+AOW)/((1.-AOW)**1.5)
  XK(IDEF,1)=FAOW*SIGZ / (WIDTHS(2) * SQRT(WIDTHS(1)))

```

Figure 4-2
Coefficient error for Configuration 101

Configuration 104 (Standard Three-Point Bend Specimen)

Figure 4-3 shows the geometry for configuration 104, Standard Three-Point Bend Specimen. The K solution for configuration 104 is coded correctly but an onscreen message is misleading to the user. The onscreen message occurs during definition of the specimen geometry as shown in Figure 4-4. The message should read *Please Note: For K solution, L is set equal to 2W, no matter what value of L is entered.* In NASCRAC, 4W in the message should be replaced by 2W.

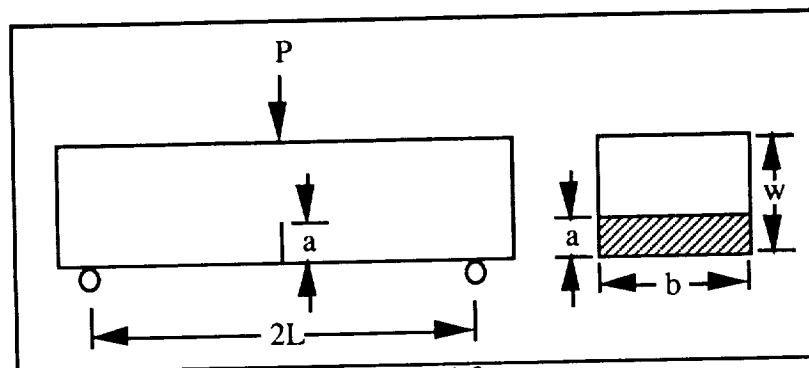
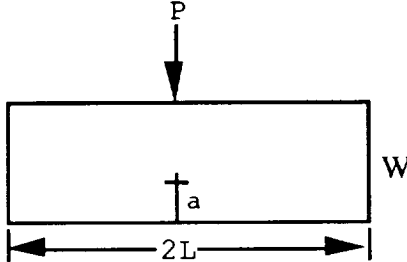


Figure 4-3
Geometry for NASCRAC Configuration 104

STANDARD 3-POINT BEND SPECIMEN		[104]

Variable Thickness:	Not Available	
Crack Position Xc:	Not Used	
Yc:	Not Used	
Crack Orientation Phi:	Not Used	
Stress Input Options :	Pin Load only; Use Equation Type 6	
J-Integral Solutions :	Available for plane stress and plane strain	
	$0.125 \leq a/W \leq 0.875$	$1 \leq n \leq 20$

		Please Note: For K Solution, L is set equal to 4W, no matter what value of L is entered.
Inputs Required: a = Crack depth; W = Width in direction of crack B = Specimen thickness; L = Specimen half length		

Enter a, W, L, and B		

Figure 4-4
Error in onscreen note

200 Series Results

Configurations 202, 204 (Center Cracked Panel, Double-Edged Cracks in a Plate)

Configurations 202 (center cracked panel) and 204 (double edged cracks in a plate) are symmetrical configurations. This symmetry led to a misinterpretation of results for linearly increasing and decreasing loads because the output K value was assumed to be the maximum value of the two crack tips. In fact, NASCRAC consistently outputs the K value for the left hand crack tip; thus, linearly increasing and decreasing loads of equal magnitude should and do produce different values. To prevent misinterpretation by inexperienced NASCRAC users, the NASCRAC documentation and written output should clearly identify which crack tip is being reported. The geometries for configuration 202 and 204 are shown below in Figures 4-5 and 4-6 respectively.

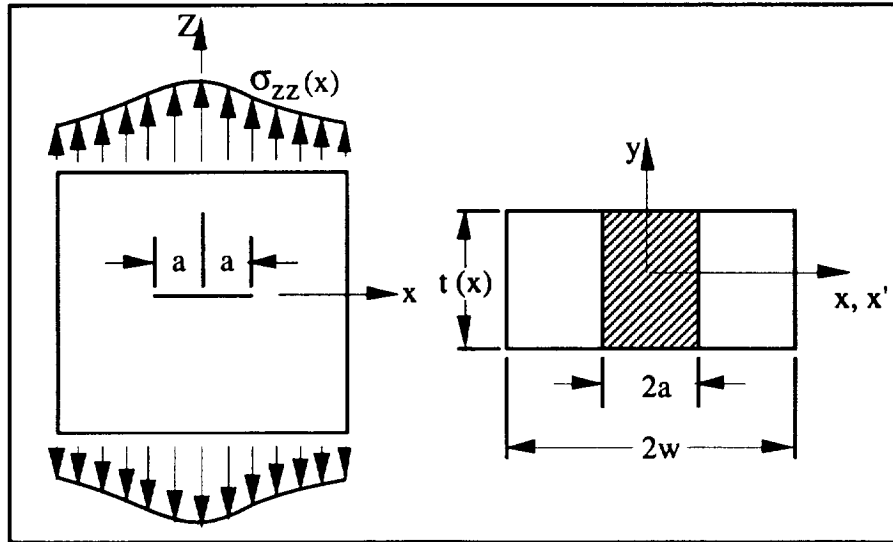


Figure 4-5
Geometry for NASCRAC Configuration 202

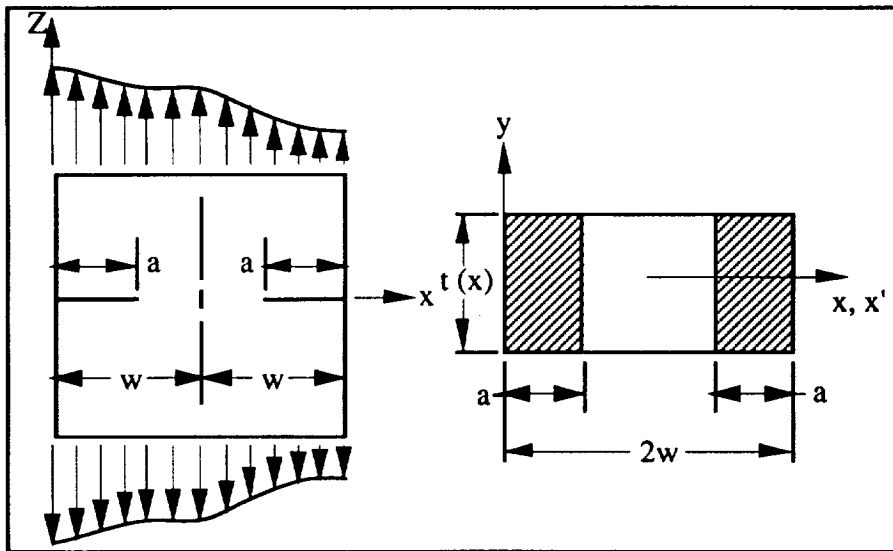


Figure 4-6
Geometry for NASCRAC Configuration 204

Configurations 205, 207 (Axial (ID,OD) Crack in a Hollow Cylinder)

The geometry for configuration 205, Axial (ID) Crack in a Hollow Cylinder, is shown in Figure 4-7. The geometry for configuration 207, Axial (OD) Crack in a Hollow Cylinder, is shown in Figure 4-8. For configurations 205 and 207 NASCRAC has two separate solutions, a uniform tension solution and a weight function solution. Uniform tension results compare well to a number of reference results. Weight function solutions are available for a limited number of inner radius to wall thickness (r_i/W) ratios; however, NASCRAC does not prevent the user from analyzing other r_i/W ratios. If an uncoded ratio is specified NASCRAC automatically uses one of its coded ratios to compute results and warns the user that the analysis was completed for a coded ratio, not the ratio specified by

the user. This approach is not erroneous but, since NASCRAC is designed to be an engineering tool, such logic increases the chances of human error. The analysis of r_i/W ratios not coded in NASCRAC (for 205, coded ratios include 1,5,10; for 207, the only coded ratio is 1) should be prevented using an error flag. This will force the analyst to bound or extrapolate his configuration using the coded solutions and will also force recognition of the assumptions used to complete the analysis.

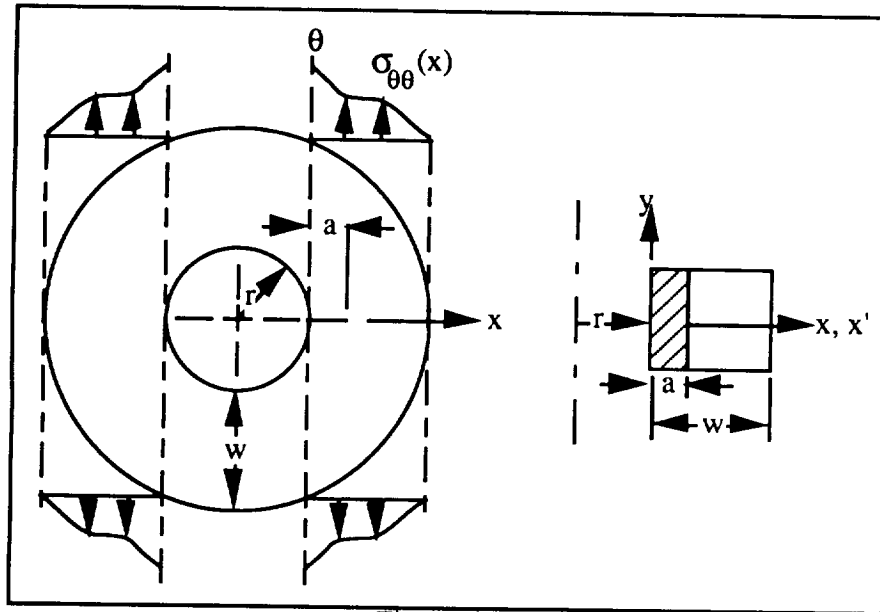


Figure 4-7
Geometry for NASCRAC Configuration 205

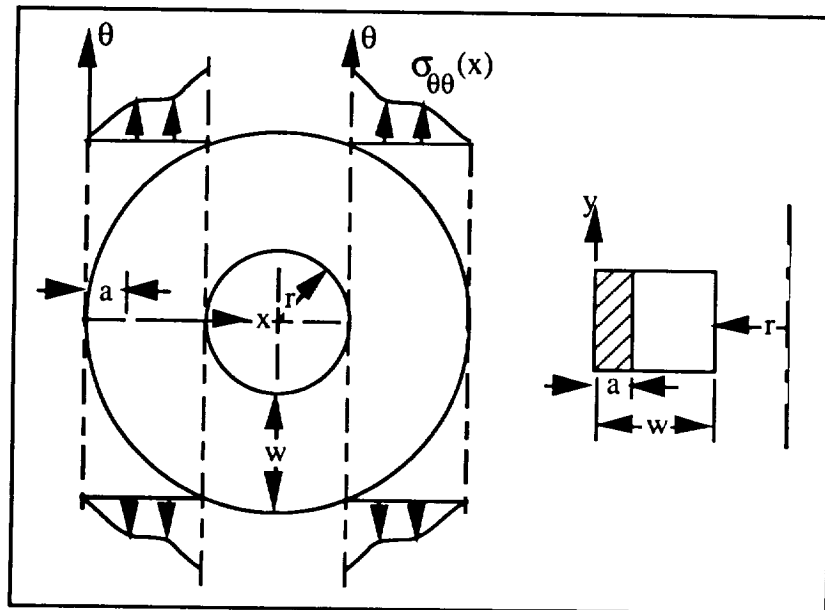


Figure 4-8
Geometry for NASCRAC Configuration 207

To illustrate the potential problem, a 205 configuration was analyzed with $r_i/W = 0.5, 1.0, 2.0, 3.0, 3.25, 4.75, \text{ and } 5.25$. For each r_i/W ratio, the cylinder wall thickness, the crack length, and the stresses on the crack plane were identical. The only variable was the inner radius of the cylinder. Results of the analysis are listed in Table 4-1; identical results were observed for $r_i/W = 0.5, 1.0, 2.0, \text{ and } 3.0$ and also for $r_i/W = 3.25, 4.75 \text{ and } 5.25$. The first set of identical results corresponds to $r_i/W = 1.0$ and the second set corresponds to $r_i/W = 5.0$. Figure 4-9 shows a condensed version of the output file for $r_i/W = 2.0$ with the r_i/W warning listed on the geometry page. The calculated results are reasonable for the r_i/W ratio ($r_i/W=1.0$) but are not necessarily reasonable for the specified r_i/W ratio ($r_i/W=2.0$), which could mislead an analyst. This conclusion is supported by Table 4-1 as the crack length increases. From Table 4-1, if the geometry of interest were $r_i/W = 3.0$ with $W = 2.0$, NASCRAC would calculate $K = 2.32$ for $a = 1.5$; however, if the cylinder radius increased slightly such that $r_i/W = 3.25$ with $W = 2.0$, then NASCRAC calculates $K = 5.34$ for $a = 1.5$. Again this discrepancy arises because NASCRAC is using its $r_i/W = 1$ solution in the first case and its $r_i/W = 5$ solution in the second case. In future NASCRAC releases, a minimum update to this solution should include this r_i/W warning on the K vs a results page as well as the geometry page. Again, as suggested above, the best resolution of this potential problem is to prevent an analyst from specifying an uncoded r_i/W configuration by including a geometry error flag.

Table 4-1
Comparison of NASCRAC 205 output for various specified r_i/W ratios

a	r_i/W						
	0.5	1.0	2.0	3.0	3.25	4.75	5.25
	K						
0.5	1.26	1.26	1.26	1.26	1.53	1.53	1.53
1.0	1.83	1.83	1.83	1.83	3.00	3.00	3.00
1.5	2.32	2.32	2.32	2.32	5.34	5.34	5.34


```

PROBLEM TITLE : g205ratioc2

-> Axial(ID) crack in a hollow cylinder 205

** WARNING : Ri/h = 2.0000
For stresses defined by Equation 1, K solution for
Ri/h = 2.00 will be used

IF solution for Ri/h = 1 will be used otherwise.

Initial Crack Dimension(1) = 0.50000
Final Crack Size = 1.50000
Crack Size Increment = 0.10000
BODY WIDTHS(1) = 2.00000
BODY WIDTHS(2) = 4.00000

TOTAL NUMBER OF TRANSIENTS ENTERED : 1
TRANSIENT NUMBER = 1
TRANSIENT TITLE = Linear
TRANSIENT TYPE = HOLD
NUMBER OF CYCLES = 1.0000E+00
DURATION = 0.0000E+00
CRACK GROWTH LAW = NOT APPLICABLE

MAXIMUM STRESS DEFINED BY EQUATION TYPE : 2 WHICH IS ...
STRESS= A0 + A1*X, A0= 1.0000E+00
A1= -5.0000E-01
MULTIPLICATION FACTOR = 1.00000E+00

Loading Block consists of the following transients -
Transient Number 1 Repeated 1 Time(s).

K VS. A SUMMARY FOR TRANSIENT # 1
A1 KMAX1 KMIN1
0.5000 1.2608 0.0000
1.0000 1.8327 0.0000
1.5000 2.3249 0.0000

```

Figure 4-9

Typical output for Configuration 205 and 207 including r_i/W warning

Configuration 208 (Through Crack from a Hole in a Finite Plate)

The geometry for configuration 208, Through Crack from a Hole in a Plate, is shown below in Figure 4-10. Results from the literature and FRANC analyses depicted a dependency of K on plate height. The results, shown in Table 4-2, suggest that K is independent of plate height for plate to width ratios (H/W) ≥ 2 . NASCRAC is in good agreement with the literature and FRANC for such ratios. However, for $H/W < 2$, NASCRAC differs from the reference solutions by 10-30%. These results suggests that the NASCRAC solution should be considered valid for $H/B \geq 2$ and caveated for $H/B < 2$. Notes in the documentation, onscreen, and in printouts should warn users that use of the solution for cases where $H/B < 2$ is marginally acceptable and should be used with caution.

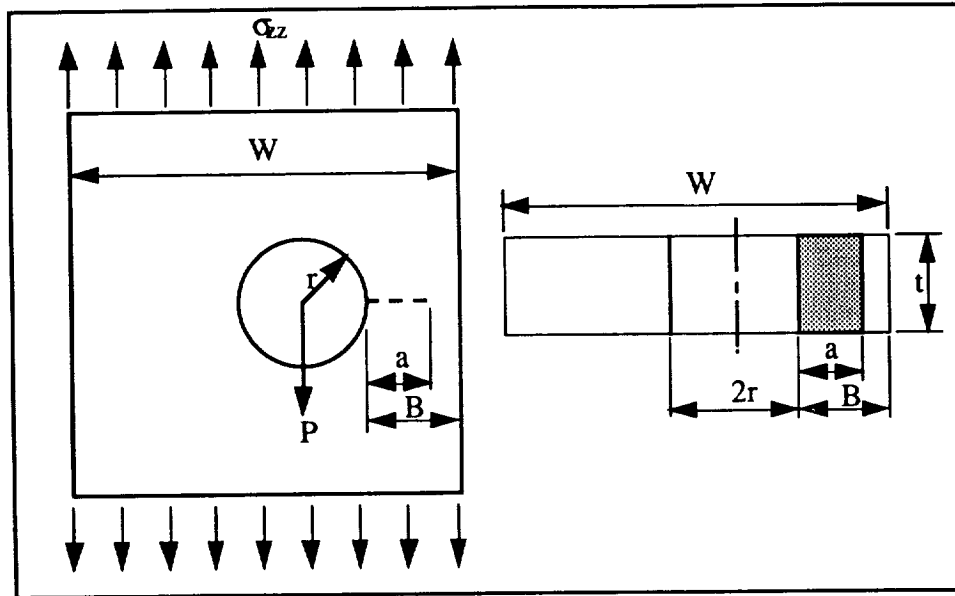


Figure 4-10
Geometry for NASCRAC Configuration 208

Table 4-2
Dependency of Configuration 208 on Plate Height to Width Ratio

H/W	a	NASCRAC	Literature	FRANC
0.5	1.0	3.31		5.05
	2.0	3.71		6.33
	4.0	5.18		9.16
1.0	1.0	3.31	3.76	3.72
	2.0	3.71	5.47	5.46
	4.0	5.18		
2.0	1.0	3.31	3.34	3.34
	2.0	3.71	3.71	3.74
	4.0	5.18	5.13	5.18
3.0	1.0	3.31	3.34	3.33
	2.0	3.71	3.69	3.71
	4.0	5.18	5.07	5.17

300 Series Results

Configurations 301, 302 (Through Crack in a Sphere, Axial Through Crack in a Cylinder)

Figures 4-11 and 4-12 show the geometries of configuration 301, Through Crack in a Sphere, and configuration 302, Through Crack in a Cylinder - Axial, respectively. No errors were found in the K solutions for configurations 301 and 302; however, three minor changes would improve the solutions. First, a note should be included in the User's Manual and onscreen stating that thin shell theory is assumed for the solutions. Secondly, the User's Manual should clearly identify which radius (inner radius) is required for input. Finally, an error flag should be included in the code to detect specified geometries which do not meet thin shell requirements.

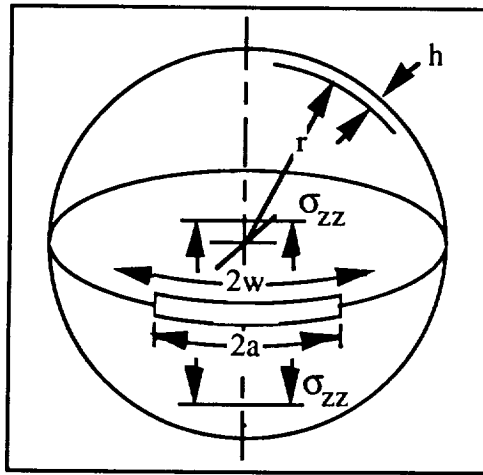


Figure 4-11
Geometry for NASCRAC Configuration 301

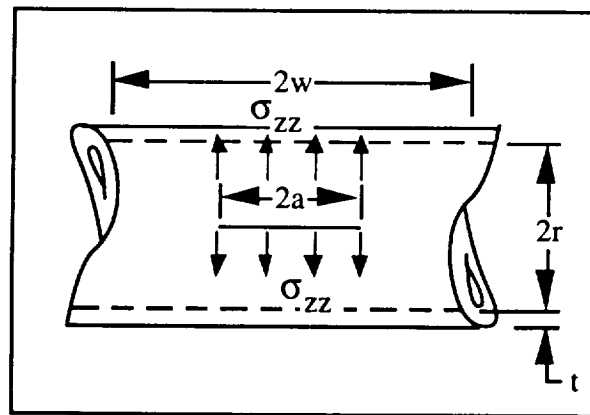


Figure 4-12
Geometry for NASCRAC Configuration 302

Configuration 303 (Through Crack in a Cylinder - Circumferential)

The geometry for configuration 303, Through Crack in a Cylinder - Circumferential, is shown below in Figure 4-13. No significant discrepancies were observed with configuration 303; however, shortcomings and errors in the documentation were discovered. First, the documentation did not clearly state that the solution was valid for the typical range of Poisson's ratio (ν). Although the solution uses $\nu = 0.3$, other values of ν ($0.0 \leq \nu \leq 0.33$) do not significantly affect the results. In the solution, Poisson's ratio is included in the shell parameter ϵ where $\epsilon = (t/R_m)^{0.5} (12(1-\nu^2))^{-0.25}$. For ν between 0.0 and 0.33, ϵ ranges from $0.537 (t/R_m)^{0.5}$ to $0.553 (t/R_m)^{0.5}$ and therefore the hardwired value is acceptable. Secondly, the NASCRAC documentation needs to clearly state that the computed K value corresponds to the midsurface of the cylinder wall. Thus, no local bending of the pressure vessel is computed. In reality, a higher K will occur at the inner or outer surface of the cylinder wall but this discrepancy should be small, especially for thin-walled cylinders. This local bending occurs even in the uniform tension case and therefore is not due to an input bending stress [10]. Finally, the NASCRAC documentation in the theory manual contains at least three typographical mistakes in the listed equations. The

mistakes are in the I_0 , C , and λ formulations. The errors and corrected equations are given in Table 4-3 below.

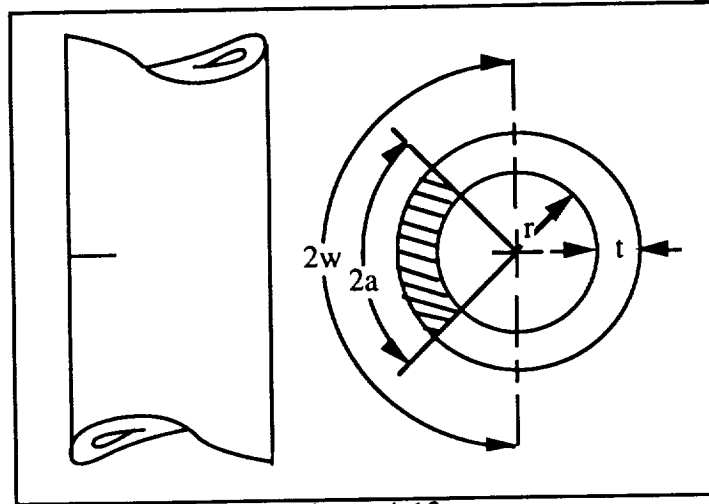


Figure 4-13
Geometry for NASCRAC Configuration 303

Table 4-3
Documentation Errors for Configuration 303

$I_0 = \alpha^2 \left[g(\alpha) + \frac{\pi C^2}{\lambda} - 2^{1.5} \right] \longrightarrow I_0 = \frac{\alpha^2}{\epsilon} \left[g(\alpha) + \frac{\pi C^2}{\lambda} - 2^{1.5} \right]$	
$C = 1 + \frac{\pi}{16} \lambda^2 - 0.0293 \lambda^2 \longrightarrow C = 1 + \frac{\pi}{16} \lambda^2 - 0.0293 \lambda^3$	
$\lambda = \frac{a}{\sqrt{rt}} \longrightarrow \lambda \sim \frac{a}{\sqrt{rt}} ; \lambda = \frac{\alpha}{2\epsilon}$	

400 Series Results

Configuration 403 (Circumferential Crack (OD) in a Hollow Cylinder)

The geometry for configuration 403, Circumferential Crack (OD) in a Hollow Cylinder, is shown in Figure 4-14 below. The coded solution in NASCRAC is limited to $0.05 \leq R_i/R_o \leq 0.95$; however, the reference solution [12] only contains results for $0.1 \leq R_i/R_o \leq 0.9$. NASCRAC permits other configurations (outside the $0.05 \leq R_i/R_o \leq 0.95$) to be input but issues a warning in the output file on the configuration page. As a minimum, this warning should also appear on the tabulated K vs a page and should be issued when R_i/R_o is outside the reference range (0.1 - 0.9). A more desirable improvement is to prohibit the user from analyzing a configuration outside the R_i/R_o range by coding an error flag in the program. Additionally, the NASCRAC solution is only valid for uniform tension. A flag detecting this limitation should be coded into the program. The curve fit solution in NASCRAC is in good agreement with a referenced weight function solution for uniform tension [14] and a referenced graphical solution [10] and is conservative compared to NASA/FLAGRO; thus, the solution is valid for $0.1 \leq R_i/R_o \leq 0.9$.

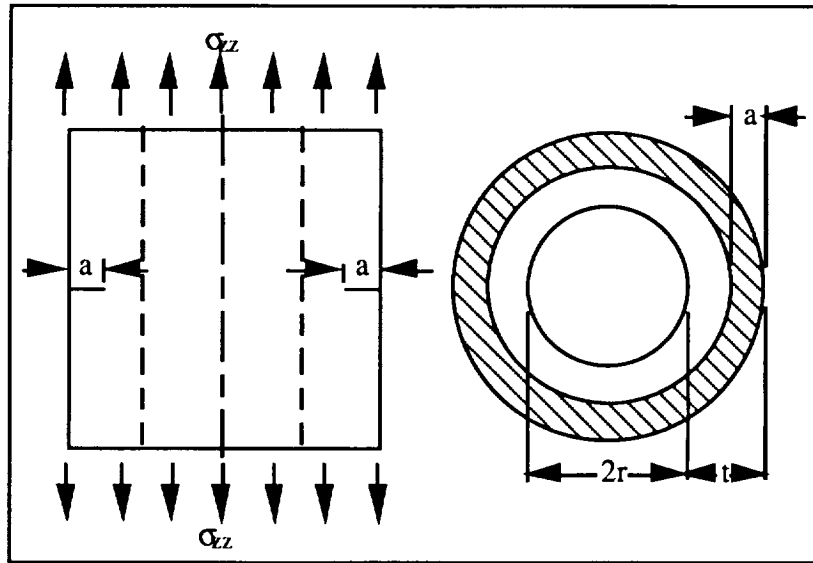


Figure 4-14
Geometry for NASCRAC Configuration 403

Configuration 404 (Edge Crack in a Solid Circular Bar)

The 404 K solution was developed by Forman and Shivakumar for NASA/FLAGRO [13]. The solution is a curve fit based on test results and a hypothetical crack front. The crack front model assumes that the crack is perpendicular to the bar at the free surface. This crack front, which results in higher K values when compared to a circular crack front whose center is at the surface of the bar, allows the crack to be specified using the crack length at the crack centerline and the radius of the bar. Figure 4-15 displays this crack front definition. The crack front equations listed in Figure 4-15, indicate that this geometry is mathematically undefined for $a/R \geq 1.0$; however, test results in reference 13 included cracks with $a/D \leq 0.6$ and the Forman-Shivakumar curve fit was calculated for $a/D \leq 0.6$. Thus, as a minimum, the NASCRAC K solution for configuration 404 should be limited to $a/D \leq 0.6$. Preferably the limit should be set to $a/D < 0.5$. To impose this limitation, an error flag should be included in the code to detect $a/D > 0.5$ and the crack geometry should be clearly defined in the User's Manual and the onscreen interface.

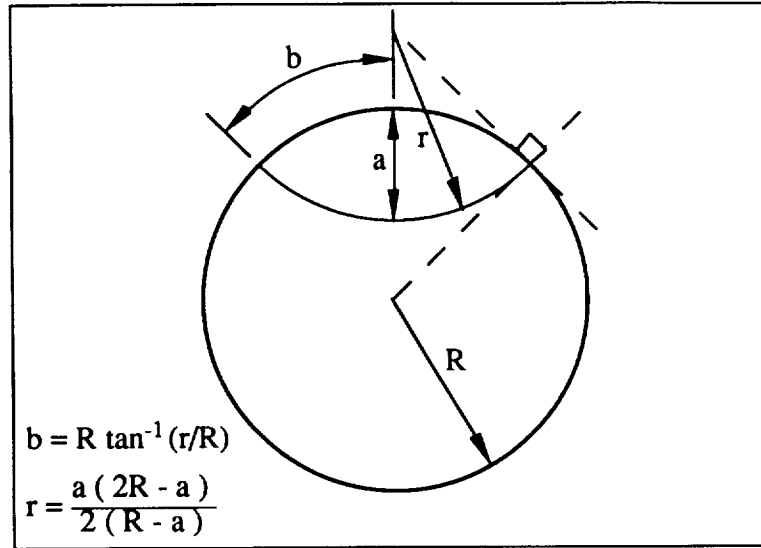


Figure 4-15
Crack Geometry for Configuration 404 (Edge Crack in a Solid Circular Bar)

In configuration 404, K varies symmetrically along the crack front. Figure 4-16 depicts this variation, which is about 10%. In Figure 4-16, NASCRAC results are identical to the Forman & Shivakumar results. The NASCRAC results corresponds to K on the centerline of the crack front. The NASCRAC documentation, onscreen information, and output files should include a note calling attention to this variation and to the fact that NASCRAC only calculates K at the midpoint of the crack, which is the minimum K along the crack front.

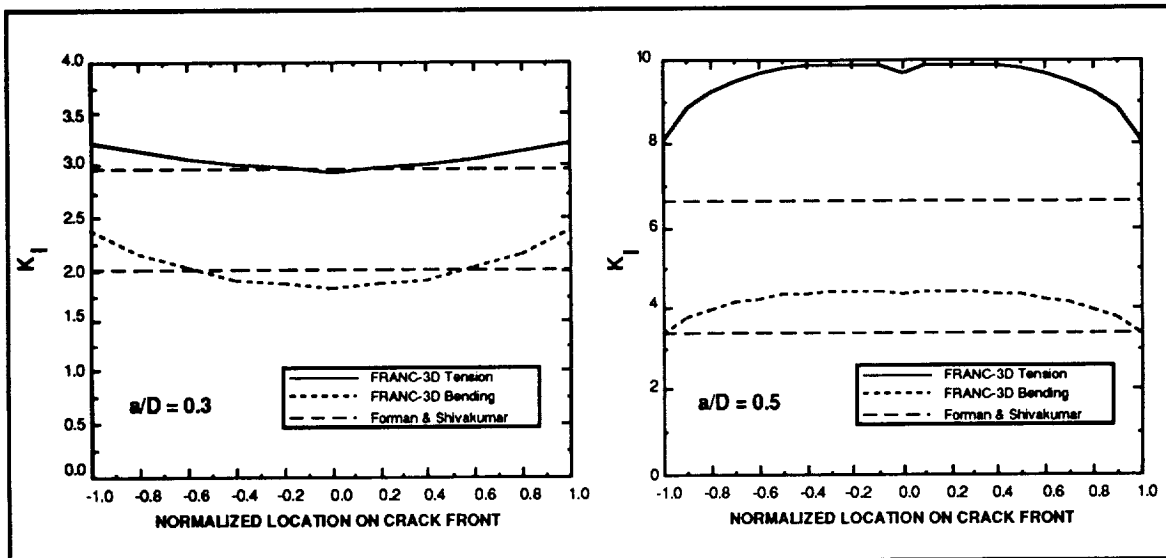


Figure 4-16
Variation in K Along the Crack Front

The case where $a/D = 0.5$ deserves special attention. From the crack front equations listed in Figure 4-15, the NASCRAC crack front would be straight for this case since $a = R$ and therefore $r = \infty$. Several literature sources were available for straight front cracks; in

particular, NASCRAC results for $a/D = 0.5$ were compared to references 1 and 11 and FRANC-3D results. Table 4-4 contains NASCRAC and reference results. Reference 1 assumes a straight front crack whereas reference 11 assumes an elliptical crack front for small a/D ratios but gradually permits the crack front to become straight as the a/D ratio increases to 0.5. From Table 4-4 it is apparent that NASCRAC agrees well with reference 11 and FRANC-3D for small a/D ratios (< 0.3) but diverges for larger a/D ratios. Compared to reference 1 (straight front crack) the NASCRAC computed K is consistently lower for all values of a/D . Additionally, FRANC-3D results appear to match the reference 1 and 11 results when $a/D = 0.5$ (straight edge cracks). These results suggest that NASCRAC may underestimate K by as much as 50% when $a/D = 0.5$, i.e., when $a = R$.

Table 4-4
Configuration 404 Results versus Reference

a	D	$\sigma_t + \sigma_b$	K NASCRAC	K Ref 11	K Ref 1	K FRANC-3D
1.0	10.0	1.0 + 1.0	2.322	2.481		
2.0	10.0	1.0 + 1.0	3.527	3.835		
3.0	10.0	1.0 + 1.0	4.950	5.793		4.77
4.0	10.0	1.0 + 1.0	6.940	9.086		
5.0	10.0	1.0 + 1.0	9.986	14.169		13.88
6.0	10.0	1.0 + 1.0	15.108	22.537		
0.5	5.0	1.0 + 0.0	0.878	0.940		
1.0	5.0	1.0 + 0.0	1.419	1.631	2.11	
1.5	5.0	1.0 + 0.0	2.104	2.562		
2.0	5.0	1.0 + 0.0	3.095	4.169	3.86	
2.5	5.0	1.0 + 0.0	4.640	6.796		
3.0	5.0	1.0 + 0.0	7.270	11.331	11.24	
1.0	10.0	1.0 + 0.0	1.241	1.329		
2.0	10.0	1.0 + 0.0	2.006	2.306	2.98	
3.0	10.0	1.0 + 0.0	2.976	3.623		2.95
4.0	10.0	1.0 + 0.0	4.377	5.895	5.46	
5.0	10.0	1.0 + 0.0	6.561	9.611		9.58
6.0	10.0	1.0 + 0.0	10.281	16.025	15.90	

The variation of K along the crack front and the inability of NASCRAC to account for this variation will lead to errors during fatigue crack growth. The calculated K value in NASCRAC is frequently the minimum K along the crack front. Thus, during fatigue crack growth, the crack front at the free surface will have a higher rate of crack growth due to a

higher K value. This variation in crack growth rate would lead to a change in crack front shape until K is uniform along the crack front. The uniform K crack front is bounded by the NASCRAC model and a straight front crack.

In summary, the NASCRAC 404 model for K vs a is valid for static checks of K where $a/D < 0.5$ if the crack front of interest adheres to the condition of intersecting the free surface perpendicularly. The geometry on which the NASCRAC curve fit model is based should be fully identified in the NASCRAC user's manual and a corresponding full explanation of the geometry should be included onscreen. Warnings should be given when applying the model to fatigue crack growth and for $a/D \geq 0.5$. Results suggest that for $a/D \geq 0.5$, NASCRAC is nonconservative by as much as 50% compared to reference results for straight crack fronts. Finally, current results show that K values for a propagated crack front whose initial shape matches the NASCRAC model are bounded by the NASCRAC model and a straight crack front model.

600 Series Results

The NASCRAC models for configurations 601 (corner crack from a hole in a plate) and 602 (corner crack from a hole in a lug) are similar. Both were derived from FLAGRO and neither incorporates a weight function. For each model, only simple loads may be applied (uniform tension and/or pin load for 601 and a pin load for 602). The V/V process for each of these models included literature sources and numerical analysis using FRANC-3D and FLAGRO. V/V results from these configurations indicate that results from NASCRAC and the references (FRANC-3D, FLAGRO, literature) are the same order of magnitude; however, NASCRAC differs from the references by 20-40% with the NASCRAC results being non-conservative. The geometries for configurations 601 and 602 and corresponding FRANC-3D boundary element models are shown in Figures 4-17 through 4-20. The dimensions in Figures 4-17 and 4-19 will be referenced in the discussion below that explains the NASCRAC results.

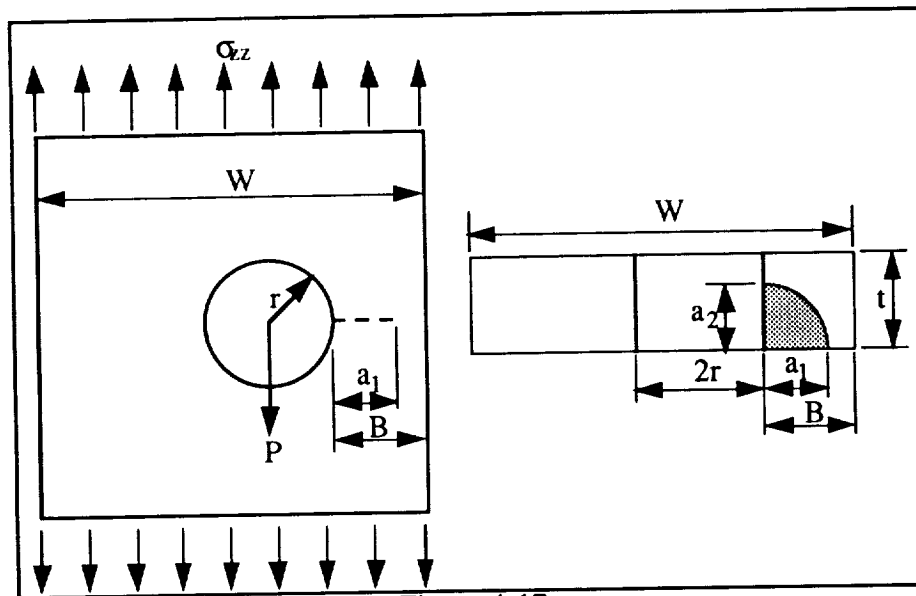


Figure 4-17
Geometry for NASCRAC Configuration 601

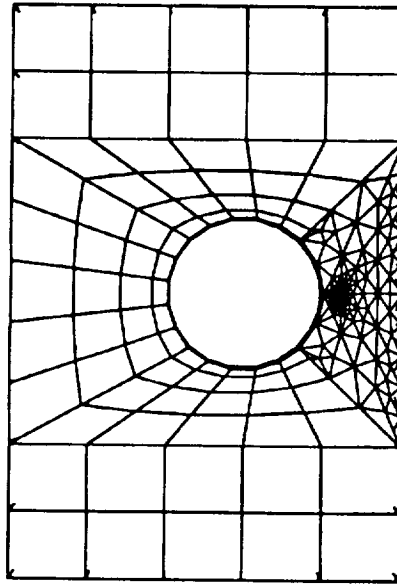


Figure 4-18
Typical FRANC-3D Boundary Element Model for Configuration 601

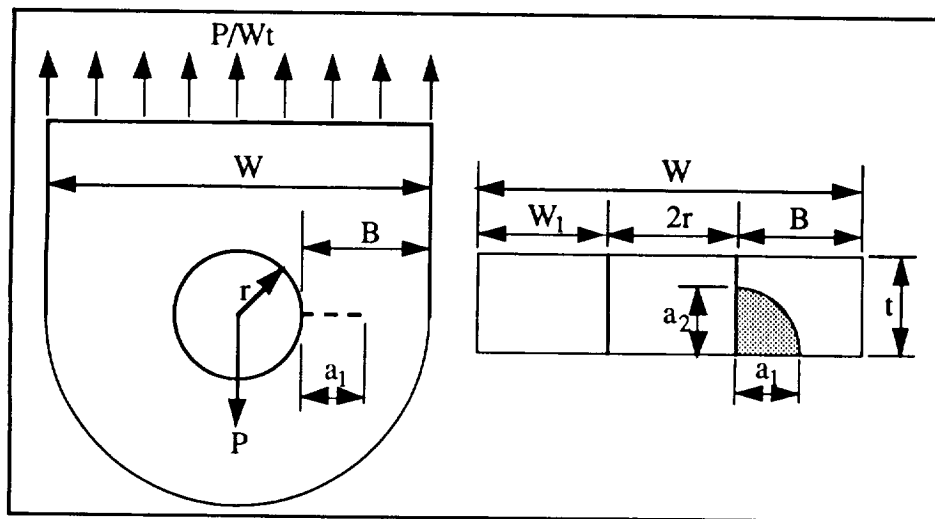


Figure 4-19
Geometry for NASCRAC Configuration 602

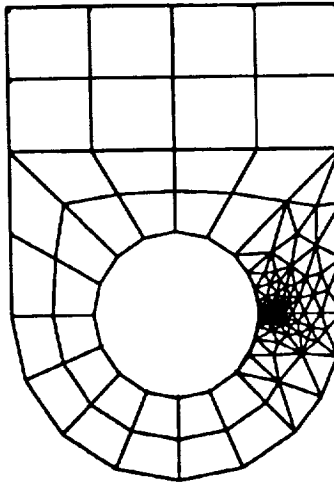


Figure 4-20
Typical FRANC-3D Boundary Element Model for Configuration 602

Figure 4-21 presents the geometry for configuration 605, Corner Crack in a Plate. NASCRAC's \bar{K} vs a capability for configuration 605 was verified and validated using the literature and FRANC. The literature included references from Newman and Raju and from Kobayashi and Enetanya for uniform tension loads. The Kobayashi paper also included linear crack pressure loads. FRANC analyses were completed for both uniform and linear loads where the linear loads were a superposition of uniform tension and bending loads across the thickness (W_2 dimension of the plate). The Kobayashi linear crack pressure load configuration and the FRANC linear load configuration are not equivalent load systems and hence cannot be compared.

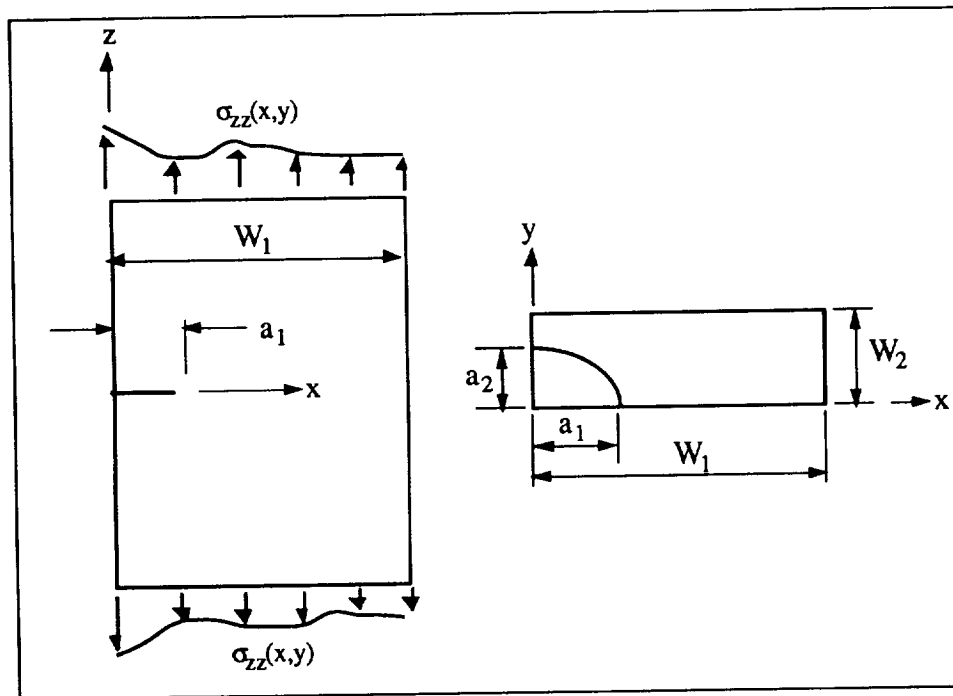


Figure 4-21
Geometry for Configuration 605: Quarter-Elliptical Corner Crack in a Plate

Configuration 601 (Corner Crack from a Hole in a Plate)

Figures 4-22 through 4-26 show results from the 601 computations. In each of these figures K's are plotted versus the corresponding crack length. Figures 4-22 and 4-24 indicate that NASCRAC does not agree with FLAGRO or FRANC when the applied load consists of a uniform stress. When the load is a pin load and the hole diameter is large compared to the crack length, NASCRAC is in agreement with the references (see Figures 4-23 and 4-25) for small crack lengths. Figure 4-26 shows results from a pin load case where the hole diameter was small compared to the crack length. These results indicate that NASCRAC may have trouble predicting the stress intensity factor along the bore of the hole (crack tip a_2). This result may be indicative of NASCRAC's handling of the stress concentration caused by the smaller radius hole or, to a lesser degree, the distribution of the load in FRANC-3D.

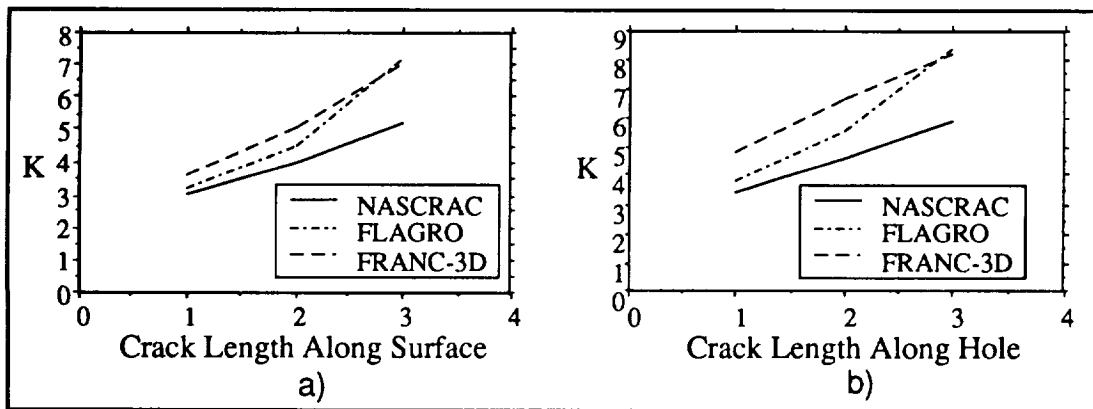


Figure 4-22

Configuration 601 in uniform tension, $a/c = 1$, $r = 4$

a) K at crack tip into plate, b) K at crack tip on surface

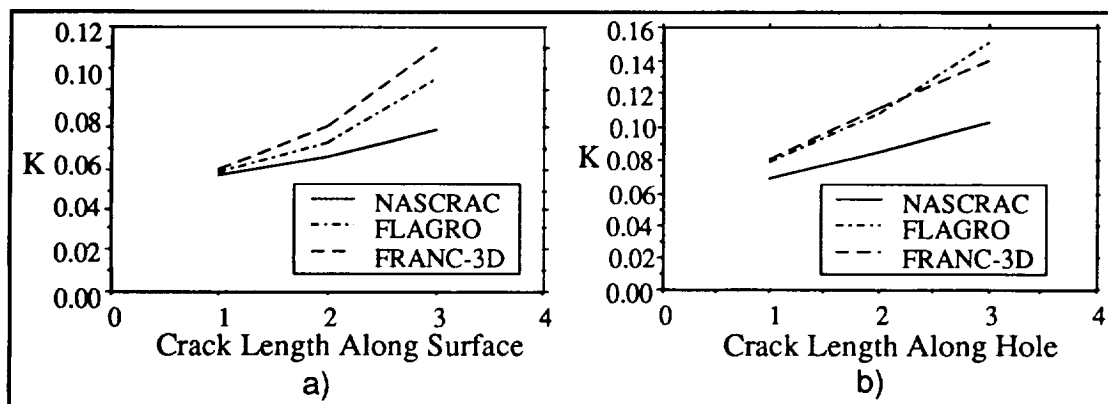


Figure 4-23

Configuration 601 with pin load, $a/c = 1$, $r = 4$

a) K at crack tip into plate, b) K at crack tip on surface

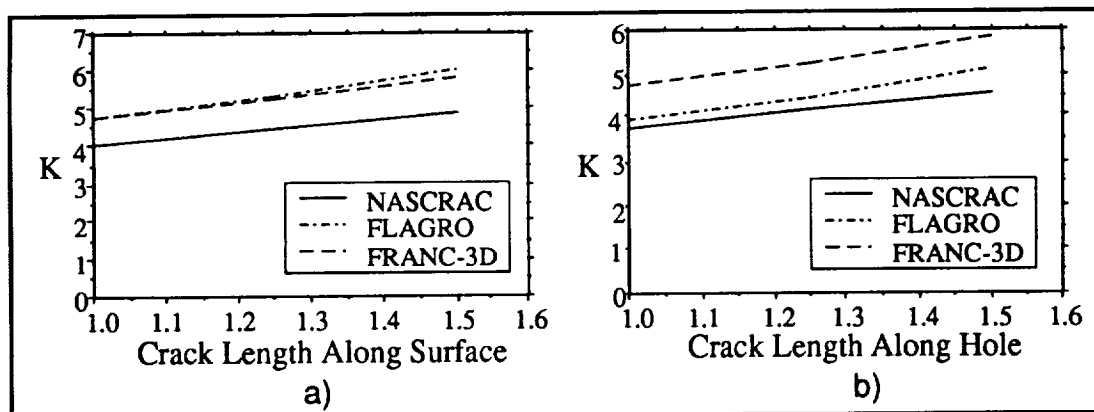


Figure 4-24
Configuration 601 in uniform tension, $a/c = 0.5$, $r = 4$
a) K at crack tip into plate, b) K at crack tip on surface

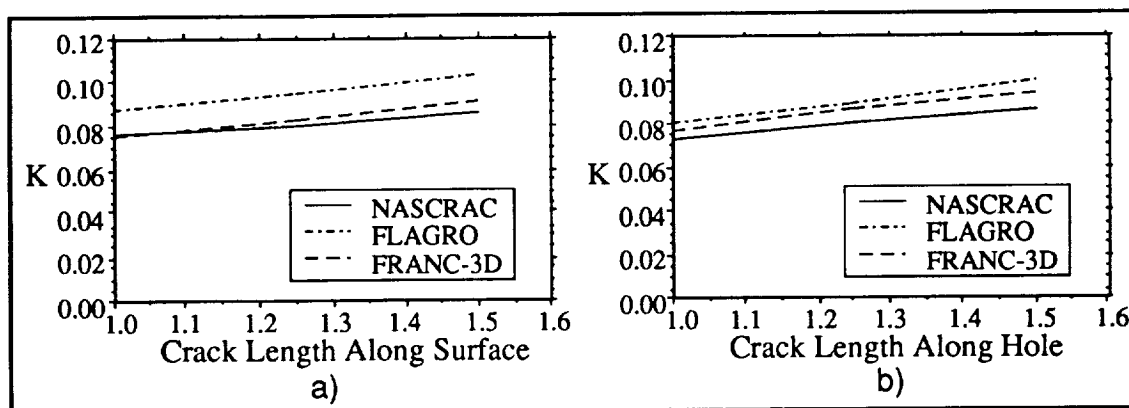


Figure 4-25
Configuration 601 with pin load, $a/c = 0.5$, $r = 4$
a) K at crack tip into plate, b) K at crack tip on surface

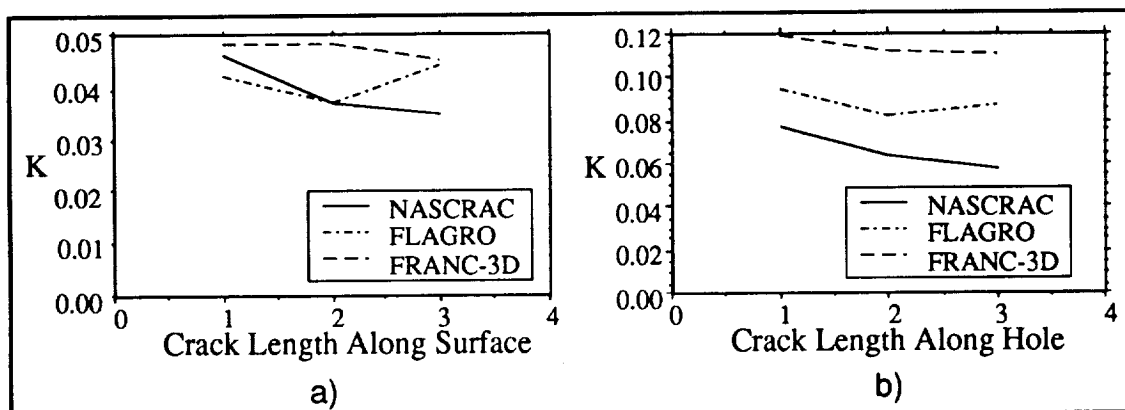


Figure 4-26
Configuration 601 with pin load, $a/c = 1$, $r = 0.5$
a) K at crack tip into plate, b) K at crack tip on surface

The trends displayed in the NASCRAC results appear to agree with FLAGRO and FRANC-3D. For example, in Figure 4-26 above, K at crack tip a_1 decreases as the crack length increases. This decrease is reflected in all three sets of the plotted results. Another trend reflected in the NASCRAC calculations is the percent change in the K values as the crack length increases. This change is reflected in the five figures above. In a majority of the cases, the absolute difference between NASCRAC and FRANC-3D is nearly constant as the crack length increases. A final trend of significance is the relative difference between a_1 and a_2 . In general, the NASCRAC differences are less than those predicted by FRANC-3D. For example, in Figure 4-23 above, the ratio of K at a_2 to K at a_1 in NASCRAC varies from 1.11 to 1.16 whereas in FRANC-3D the ratio varies from 1.17 to 1.33.

The differences between NASCRAC and FLAGRO were unexpected since the NASCRAC model was adapted from FLAGRO. A combination of two factors contribute to these differences. The first factor is a minor error in the NASCRAC source code. This error is displayed in the source code listing in Figure 4-27. FLAGRO uses 2B in the denominator of the highlighted line whereas NASCRAC uses W. If $2B = W$, which is the case for a centered hole, the error disappears. In a trial run, by changing W in NASCRAC to 2B, the computed K at tip a_2 increased from 3.40 to 3.47 and the computed K at tip a_1 increased from 3.05 to 3.12 for $B = 8.0$ and $W = 12.0$. The second factor that causes a difference between NASCRAC and FLAGRO is NASCRAC's calculation of an RMS averaged K at each crack tip using Gaussian quadrature. RMS averaging computes the K of interest by summing weighted values of K from the entire crack surface. FLAGRO, conversely, directly calculates the two K's (one at 0 degrees and one at 80 degrees) using equations identical to those in NASCRAC other than the minor error shown in Figure 4-27. Based on the FRANC-3D results, the applicability of the RMS logic in NASCRAC may not be acceptable even though the logic is valid. One final difference can be documented between NASCRAC and FLAGRO: FLAGRO accepts bending loads but NASCRAC, when adapting the solution, omitted bending loads and only permits uniform tension and pin loads.

NASCRAC	
FUNCTION	CC02 (PHI)
.	
Y	= D/W
V	= A/T
XL	= .5*PI*SQRT(V) * (D+C) / (W-C)
FW	= SQRT (SIN (BETA) / (BETA * COS (XL) * COS (.5*PI*Y)))
.	
RETURN	
END	
FLAGRO	
SUBROUTINE	SICC02 (MODE, LOCN, CREMEN, SMIN4, SMAX4, SYLD1, CAYC1,
&	A, AOC, NSQUAN, IHDSQ, META, SR, DELTAK, CAYMAX,
&	F0, F1, F2, F3, Q, NJOB, NETMSG, IACMSG, IYZMSG, *, *)
.	
GWCOEF	= (DSIN (BETA) / BETA) / DCOS (PIOVR2 * D/W)
GW	= DSQRT (GWCOEF / DCOS (PIOVR2 * DSQRT (AOT) * (D+C) / (2D0 * B - C)))
.	
RETURN	
END	

Figure 4-27

CC02 Source Code in NASCRAC and FLAGRO Highlighting Difference in Codes

Configuration 602 (Corner Crack from a Hole in a Lug)

NASCRAC's K vs a capability for configuration 602, corner crack from a hole in a lug, calculates stress intensity factors of the same order of magnitude as FLAGRO and FRANC-3D. However, the NASCRAC values are significantly non-conservative (by 20-35% for large diameter holes and 50-100% for small diameter holes) compared to FRANC-3D and slightly less than the FLAGRO results, even though the NASCRAC solution was adapted from FLAGRO. This slight discrepancy is caused by two factors: 1) NASCRAC's calculation of an RMS averaged K at each crack tip using Gaussian quadrature as compared to FLAGRO's direct calculation of K at specific angles (0 degrees, 80 degrees) along the crack front, and 2) a typographical error in the equation for G_0 in the function *SICC03*. This error, which is simply a transposition of two digits, is shown in Figure 4-28.

NASCRAC	FLAGRO
<pre> FUNCTION CC03(PHI) . F0Z=0.7071+Z*(.7584+Z*(.3415+Z*(.642+.9196*Z))) F1Z=Z*(.078+Z*(.7588+Z*(-.4293+Z*(.0644+Z*.651)))) G0=F0Z/DS . F0=(0.5*G0*Y + G1)*GW CC03=F0 . RETURN END </pre>	<pre> SUBROUTINE SICC03(MODE, LOCN, CREMEN, SMIN4, SMAX4, SYLD1, CAYC1, & A, AOC, NSQUAN, IHDSQ, META, SR, DELTAK, CAYMAX, & F0, F1, F2, F3, Q, NJOB, NETMSG, IACMSG, IYZMSG, *, *) . CAPG0 = (.7071D0 + Z*(.7548D0 + Z*(.3415D0 + Z*(.642D0 + Z*.9196D0)))) / DENOM . RETURN END </pre>

Figure 4-28

CC03 Source Code in NASCRAC and FLAGRO Highlighting Difference in Codes

Figures 4-29 through 4-31 display plots of K values versus the corresponding crack lengths for configuration 602. In all cases, the applied pin load was 1 lbf. The figures show that NASCRAC and FLAGRO are in better agreement than they were for configuration 601. Only in the case of the small radius hole (Figure 4-31) is there appreciable difference at the crack tip along the bore of the hole (crack tip a_2). This is probably a result of FLAGRO's point solution versus NASCRAC's averaged solution. In all cases, NASCRAC is non-conservative compared to FRANC-3D. This non-conservatism increases as the crack

length increases and is more pronounced at crack tip a_2 . Since FRANC-3D is a refined finite element program adept at handling the stress fields around the hole, the FRANC-3D results provide a higher level of confidence.

The 602 results plotted in Figures 4-29, 4-30, and 4-31 show that relative differences in K for various crack lengths are similar in NASCRAC compared to FRANC-3D and FLAGRO. For example, in Figure 4-29a (crack tip a_1) the percent increase in K from $a_1 = 1$ to $a_1 = 2$ is 23% in NASCRAC compared to 26% for FLAGRO and 33% for FRANC-3D. Similarly, for crack tip a_2 (Figure 4-29b), the percent increase in K from $a_2 = 1$ to $a_2 = 2$ for NASCRAC is 33% compared to 37% for FLAGRO and 36% for FRANC-3D.

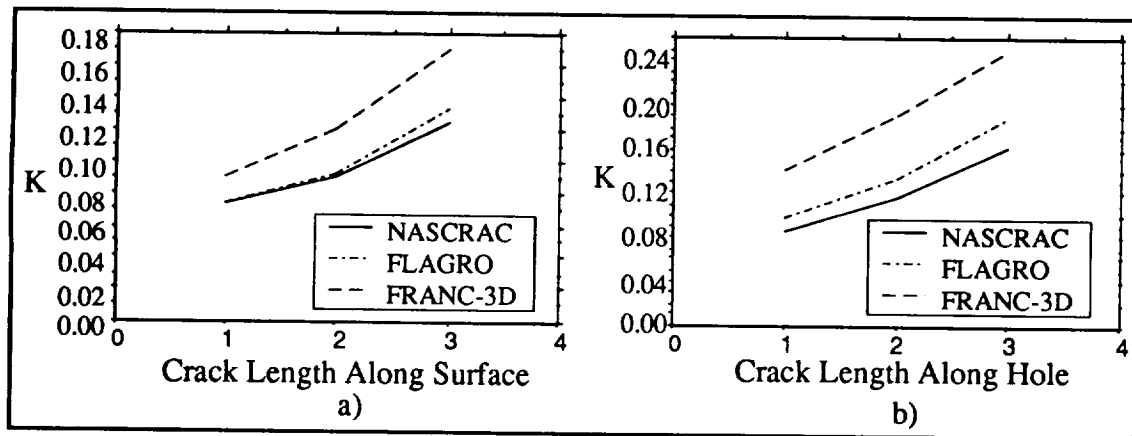


Figure 4-29
Configuration 602 with pin load, $a/c = 1$, $r = 4$
a) K at crack tip into plate, b) K at crack tip on surface

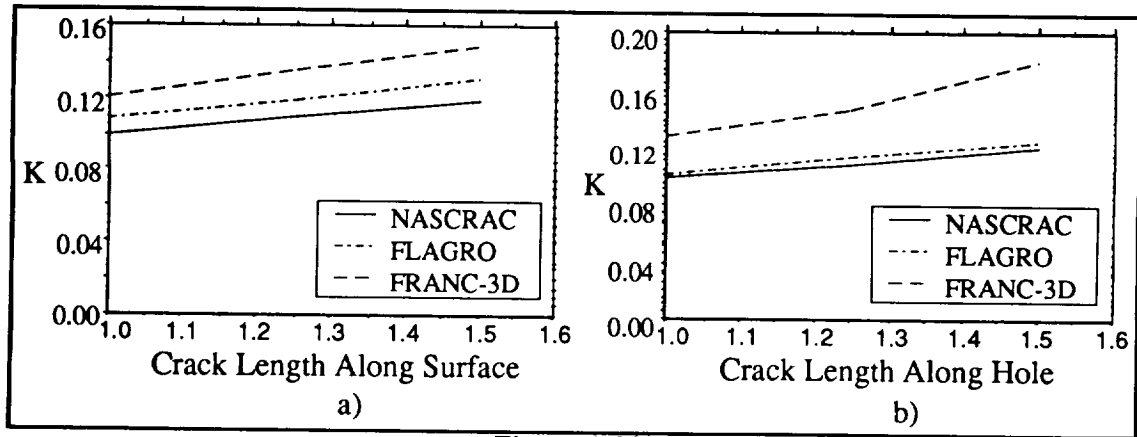


Figure 4-30
Configuration 602 with Pin Load, $a/c = 0.5$, $r = 4$
a) K at crack tip into plate, b) K at crack tip on surface

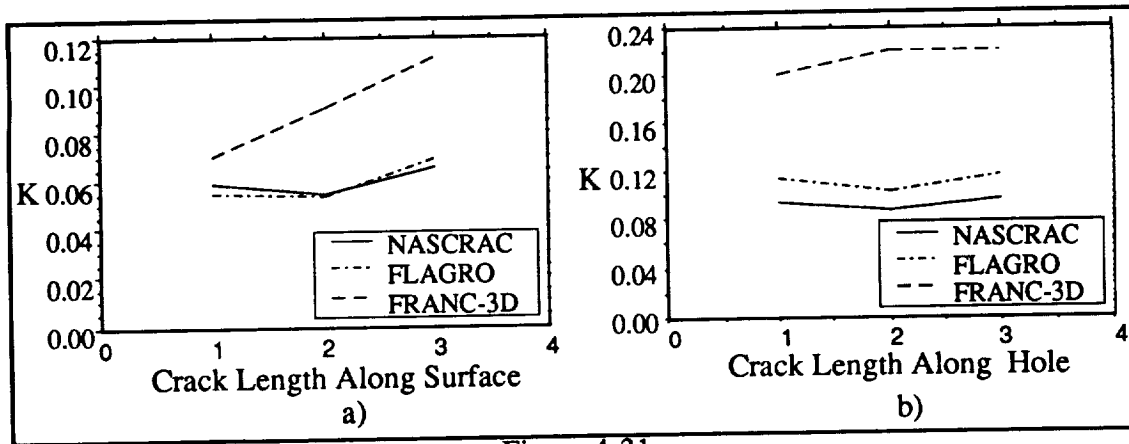


Figure 4-31
Configuration 602 with Pin Load, $a/c = 1$, $r = 0.5$
a) K at crack tip into plate, b) K at crack tip on surface

Configuration 605 (Quarter Elliptical Corner Crack in a Plate)

605 V/V results are presented in Figures 4-32 through 4-34. Each of the figures presents three cases. Case 1 consists of $W_1 = 20.0$ and $W_2 = 2.0$. Case 2 consists of $W_1 = 10.0$ and $W_2 = 0.8$. The final geometry, case 3, consists of $W_1 = 10.0$ and $W_2 = 0.2$. Figure 4-21 depicts the definition of W_1 and W_2 . Figure 2 displays plots of K vs a/W from NASCRAC, FLAGRO, and Newman and Raju for three different crack geometries subjected to uniform tension. These results indicate that NASCRAC calculates reasonable values of K for uniform tension loads. In case 1 where NASCRAC's K value at a_1 is non-conservative, the maximum difference is less than 15%. As the crack becomes smaller (cases 2 and 3), the difference between NASCRAC and the references becomes smaller. For K at a_2 , NASCRAC is consistently conservative. NASCRAC does issue a warning when a_2/W_2 exceeds 0.6 which states that the accuracy limitations of the solution have been exceeded; thus, the non-conservative results for K at a_1 occur beyond the limitations of the solution. The actual warning issued is for a_1/W_1 but this warning is incorrect. The warning should reference a_2/W_2 for the cases studied.

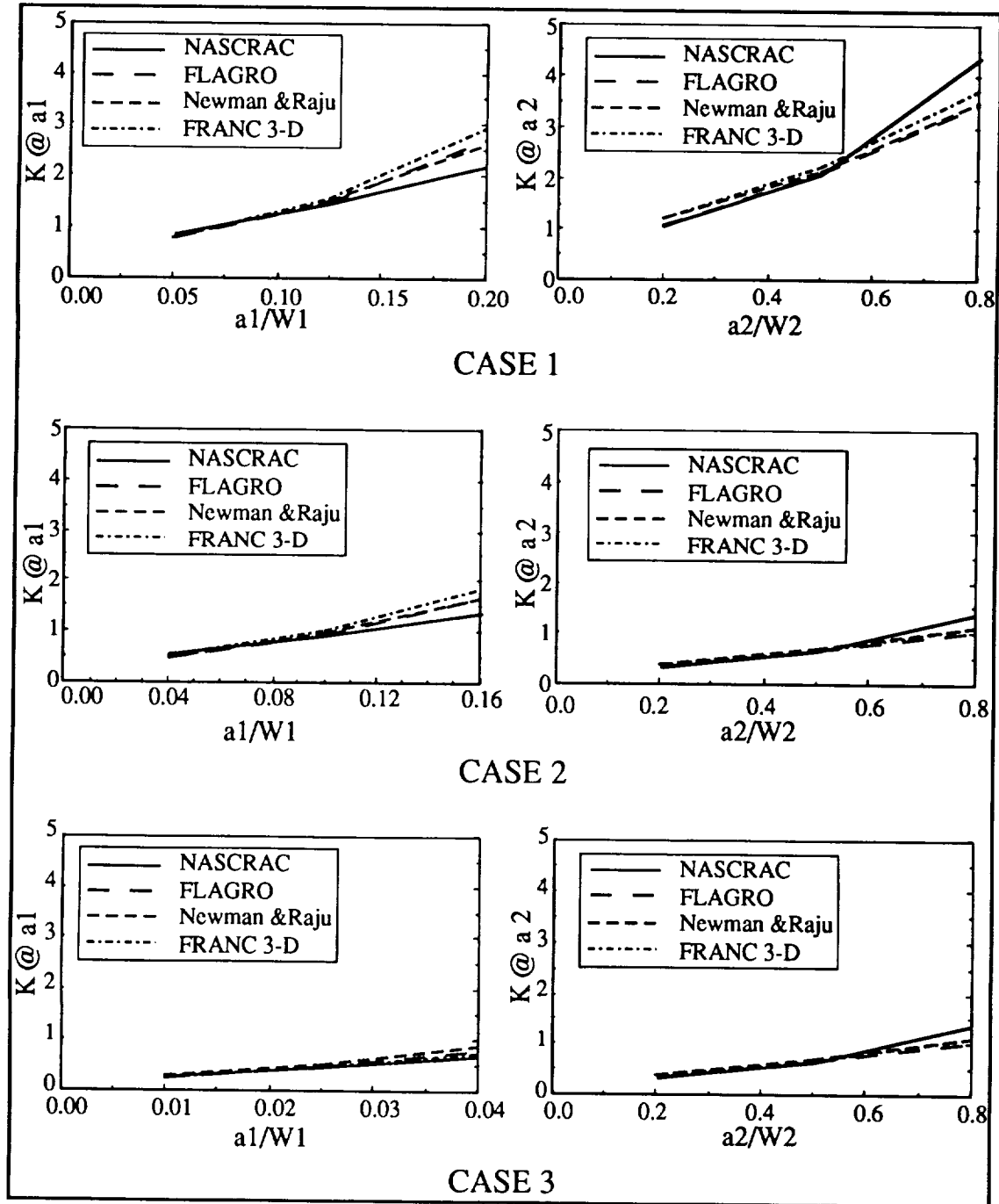


Figure 4-32
Uniform Tension Load Results from NASCRAC and References

Figure 4-33 presents K vs a/W results from NASCRAC, FLAGRO, and Newman and Raju for the three crack geometries subjected to a linear load across the thickness of the plate. NASCRAC is non-conservative for small cracks (e.g., $a_2/W_2 < 0.25$), within $\pm 40\%$ for $0.25 < a_2/W_2 < 0.6$, and conservative for larger cracks.

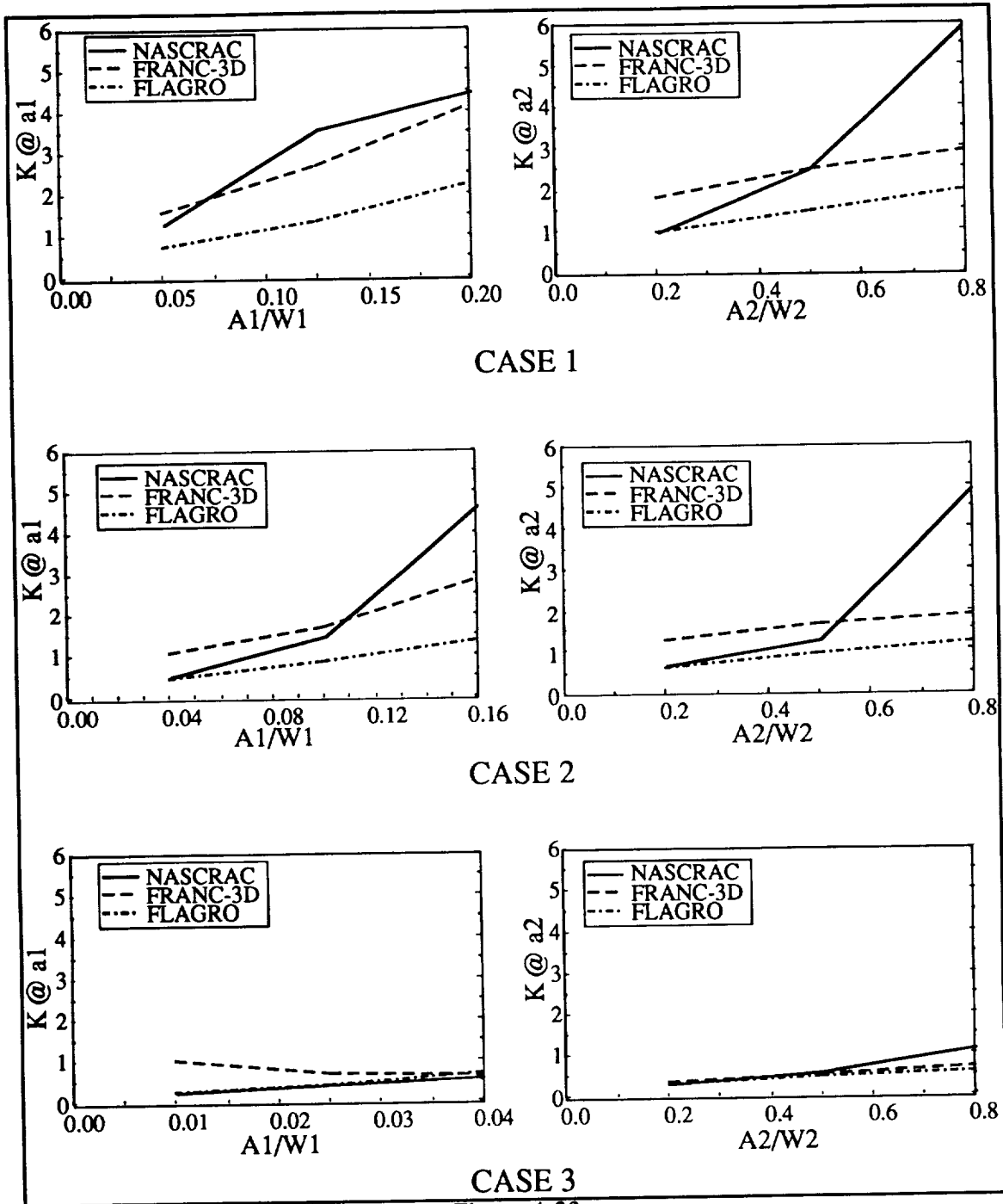


Figure 4-33
Linear Load Results from NASCRAC and References

Figure 4-34 presents K vs a/W results from NASCRAC and Kobayashi for the three crack geometries subjected to a linear crack face pressure. This figure indicates that reasonable agreement between NASCRAC and Kobayashi exists for this loading at a_1 but not at a_2 .

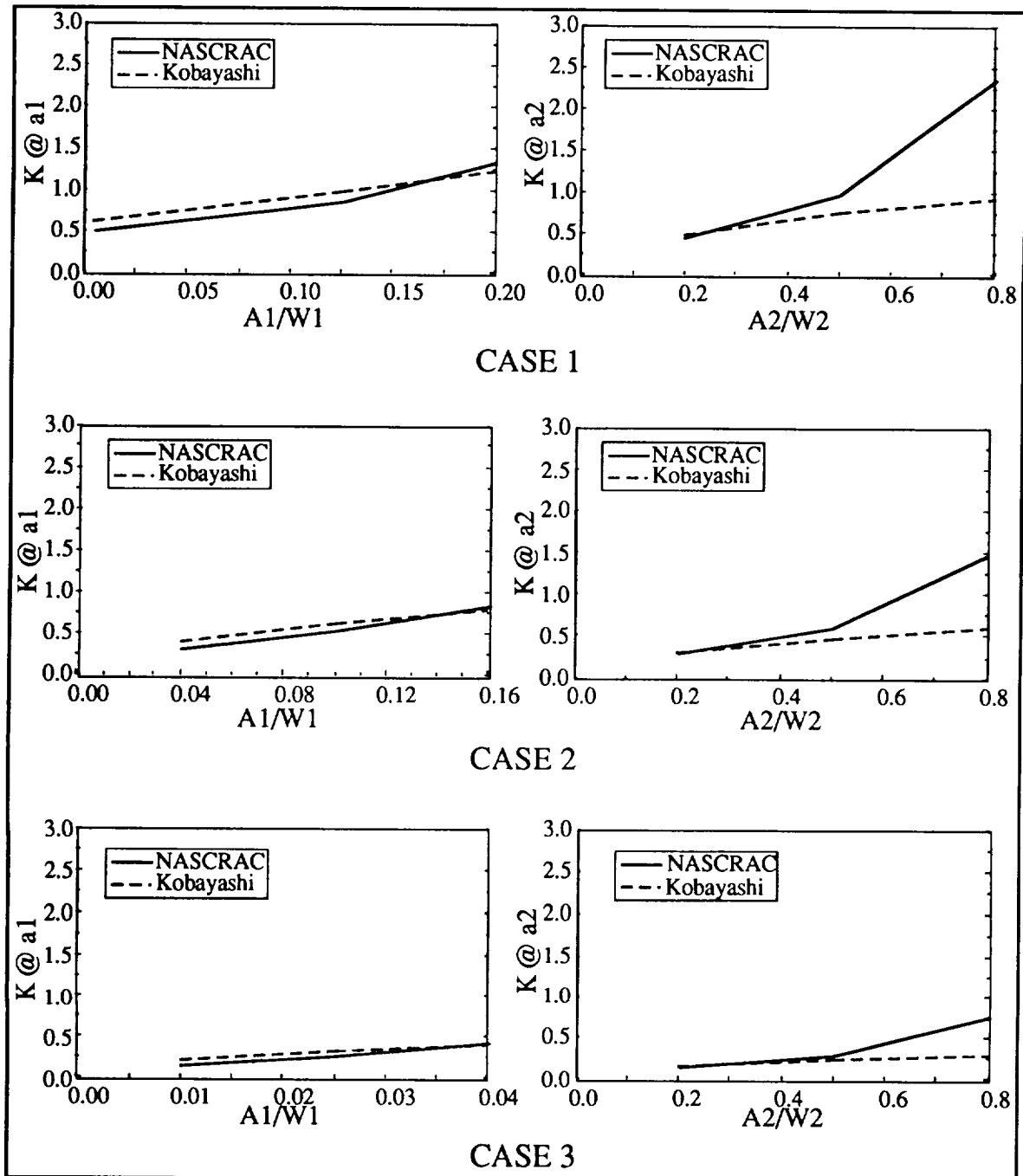


Figure 4-34
Linear Crack Face Pressure Results from NASCRAC and Kobayashi

In summary, 605 appears acceptable for uniform tension loads. For non-uniform loads, analysts can expect differences as high as 50% (both conservative and non-conservative) for specific configurations and hence should use the non-uniform load results from configuration 605 with caution. These results described above strongly suggest that further refinement is required before configuration 605 is acceptable as a valid capability for non-uniform loadings.

700 Series Results

The 700 series K vs a solutions in NASCRAC are based on the same weight function. This function was originally developed for configuration 703, a semi-elliptical (circumferential) surface crack in a cylinder. NASCRAC and reference results were in agreement for both configurations 703 and 705, a semi-elliptical surface crack in a sphere. The only problem related to these configurations was the potential for a through crack to develop without detection by NASCRAC. Results for configurations 702 and 704 are discussed below. These two configurations did not agree with the references, especially at the surface crack tip (a_2).

Configuration 702 (Semi-Elliptical Surface Crack in a Plate)

Figure 4-35 displays the geometry for configuration 702, Semi-Elliptical Surface Crack in a Plate. Several literature sources were available for the analysis of this configuration. Additionally, unpublished results from a round-robin study conducted by NASA/MSFC were available. The primary literature source was reference 7, a Raju and Newman paper describing an empirical K equation for surface cracks. The results, shown in Figures 4-36 through 4-43, indicate that the NASCRAC K model at the crack tip into the plate (crack tip a_1) is valid for the case of uniform tension (Figure 4-36). Figures 4-37 through 4-39 and Figure 4-43 indicate that the NASCRAC model for crack tip a_1 is valid in bending for crack tip to thickness ratios $a_1/t \leq 0.5$. These same figures show that NASCRAC differs from reference 7 at a_1 for bending when $a_1/t > 0.5$. For these cases, the reference values are believable because crack tip a_1 is in a region of compressive stresses and hence a reduced or negative K value is expected. The trends shown by NASCRAC for the bending cases appear reasonable. As the crack tip extends into the region of compressive stress, the value of K is less. Additionally, as the crack becomes more circular (a/c increases) the value of K at a_1 decreases. The combined bending and tension curves in Figures 40 through 42 show similar trends for crack tip a_1 , i.e., agreement between NASCRAC and reference 6 is reasonable for small a_1/t ratios but more disagreement occurs as a_1/t approaches 0.8. For crack tip a_2 , along the surface of the plate, NASCRAC was consistently non-conservative versus the references for both bending and combined bending and tension (Figures 4-40 through 4-42). NASCRAC also exhibited an unexpected trend for the cases of linear and non-linear bending, as shown in Figures 4-37 through 4-39 and 4-41. In these figures the K value at a_2 (along the surface) decreased as the crack length increased. This result is unexpected because this region incurs the maximum tensile stress.

RMS averaging causes the disagreement between NASCRAC and the references. RMS averaging computes K by summing weighted values of K over the entire crack surface. Thus, if part of the crack lies in a region of compressive or reduced tensile stresses, the averaged value of K at the crack tip of interest is less than a point calculation of the same K . This situation occurs in configuration 702 when bending loads are applied. At crack tip a_1 , which is the tip into the plate, K should decrease as a_1 becomes large, i.e., as a_1 propagates into the region of compressive stress. This behavior is observed in the Raju and Newman results plotted in Figures 4-37 through 4-43. As a_1 propagates into the compressive or reduced tensile (for combined bending and tension) region, NASCRAC does a poor job of following the Raju and Newman results because the NASCRAC computed K value is being influenced by the tensile stresses near the surfaces of the crack. Converse logic applies to crack tip a_2 . Here, the crack tip remains in a region of high tensile stress and thus K should increase in value as the crack length increases. This behavior can be seen in the Raju and Newman results plotted in Figures 4-37 through 4-43. These same figures show that the NASCRAC computed K at a_2 begins to flatten out or decrease with increasing crack length. This unexpected trend in the NASCRAC results is caused by the influence of compressive stresses in the a_1 region of the crack surface.

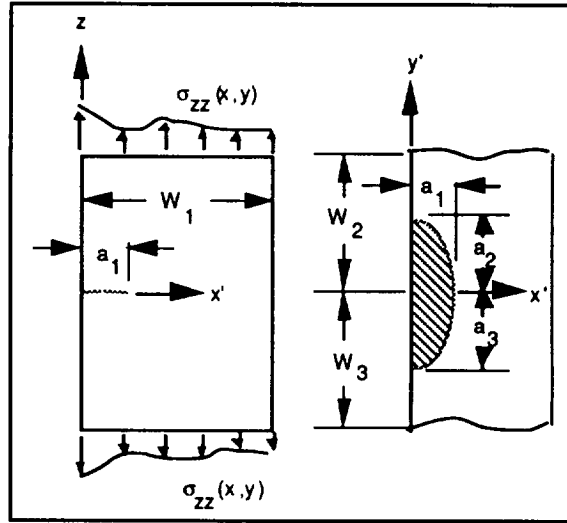


Figure 4-35
Geometry for Configuration 702

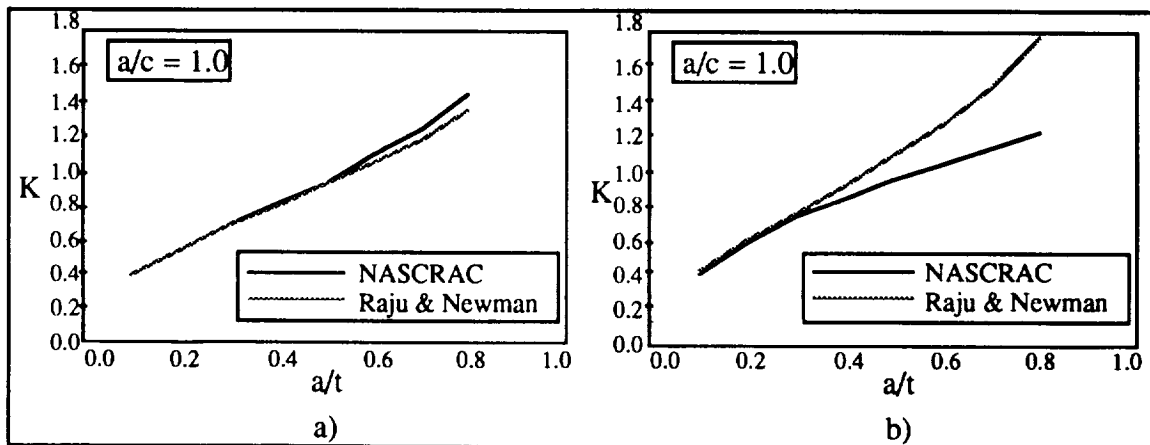


Figure 4-36
Configuration 702 in uniform tension
a) K at crack tip into plate b) K at crack tip on surface

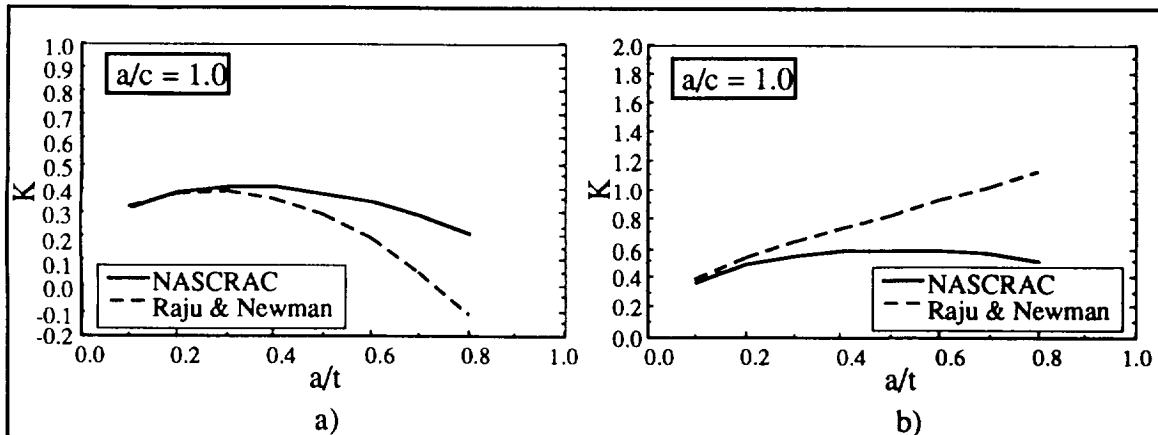


Figure 4-37
Configuration 702 in bending for $a/c = 1.0$
a) K at crack tip into plate b) K at crack tip on surface

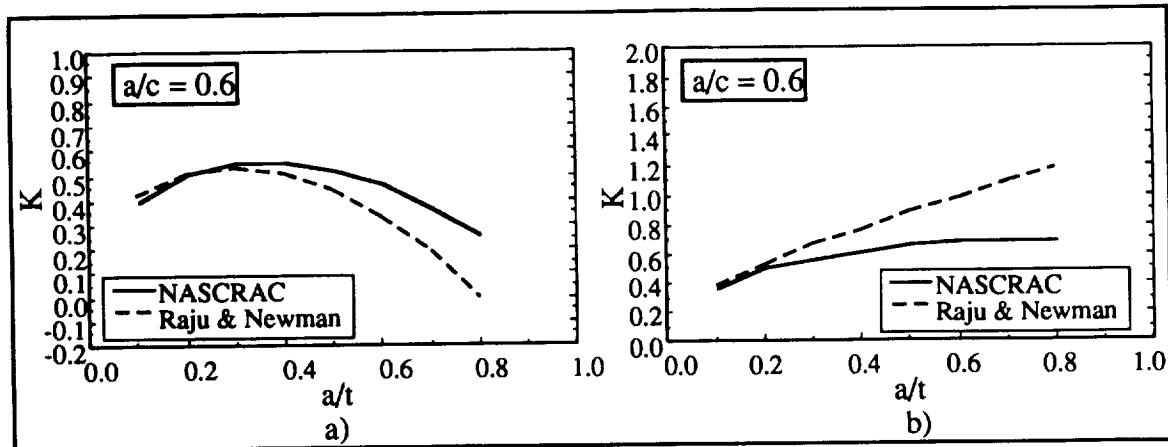


Figure 4-38
Configuration 702 in bending for $a/c = 0.6$
a) K at crack tip into plate b) K at crack tip on surface

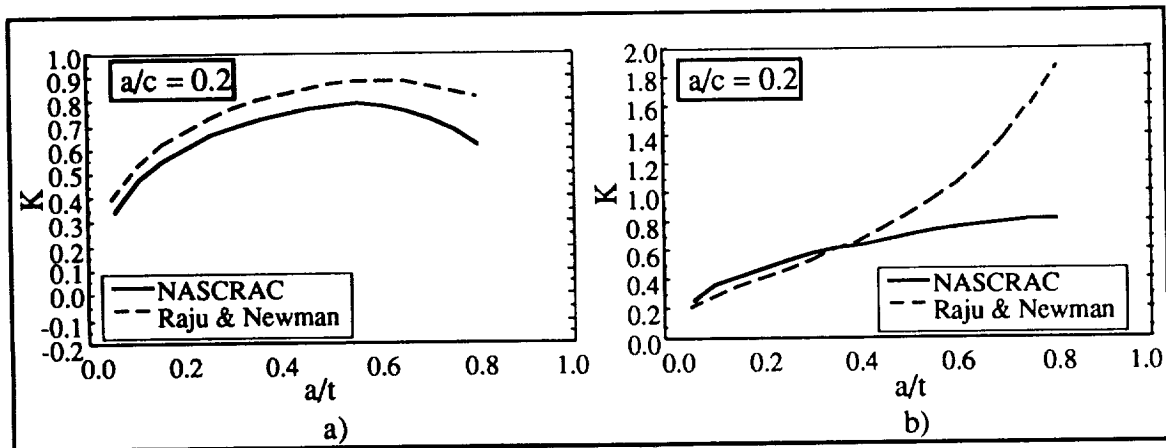


Figure 4-39
Configuration 702 in bending for $a/c = 0.2$
a) K at crack tip into plate b) K at crack tip on surface

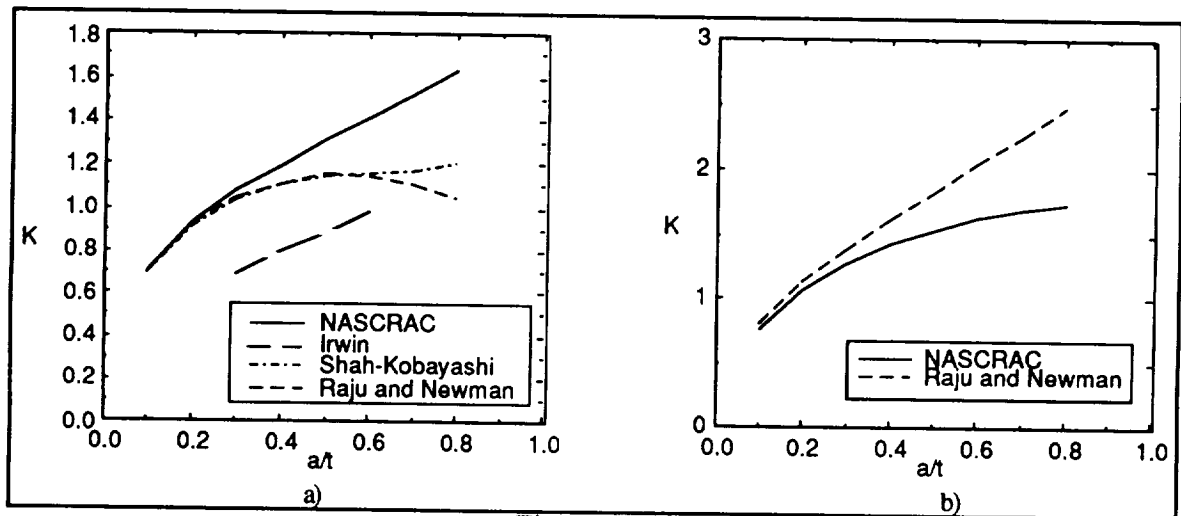


Figure 4-40
Configuration 702 in combined bending and tension for $a/c = 1$
a) K at crack tip into plate, b) K at crack tip on surface

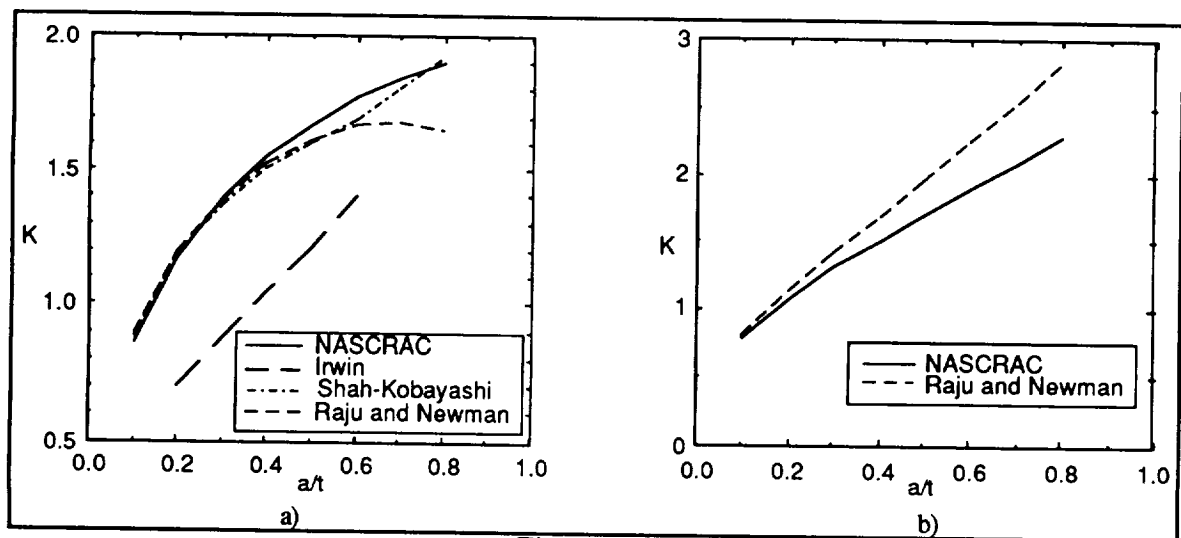


Figure 4-41
Configuration 702 in combined bending and tension for $a/c = 0.6$
a) K at crack tip into plate, b) K at crack tip on surface

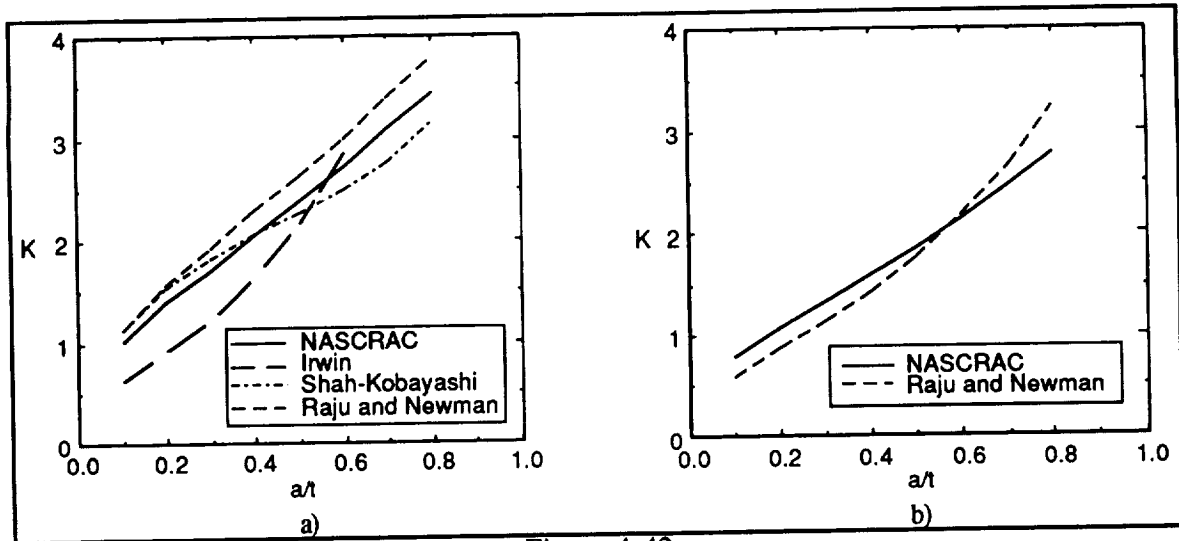


Figure 4-42
Configuration 702 in combined bending and tension for $a/c = 0.2$
a) K at crack tip into plate, b) K at crack tip on surface

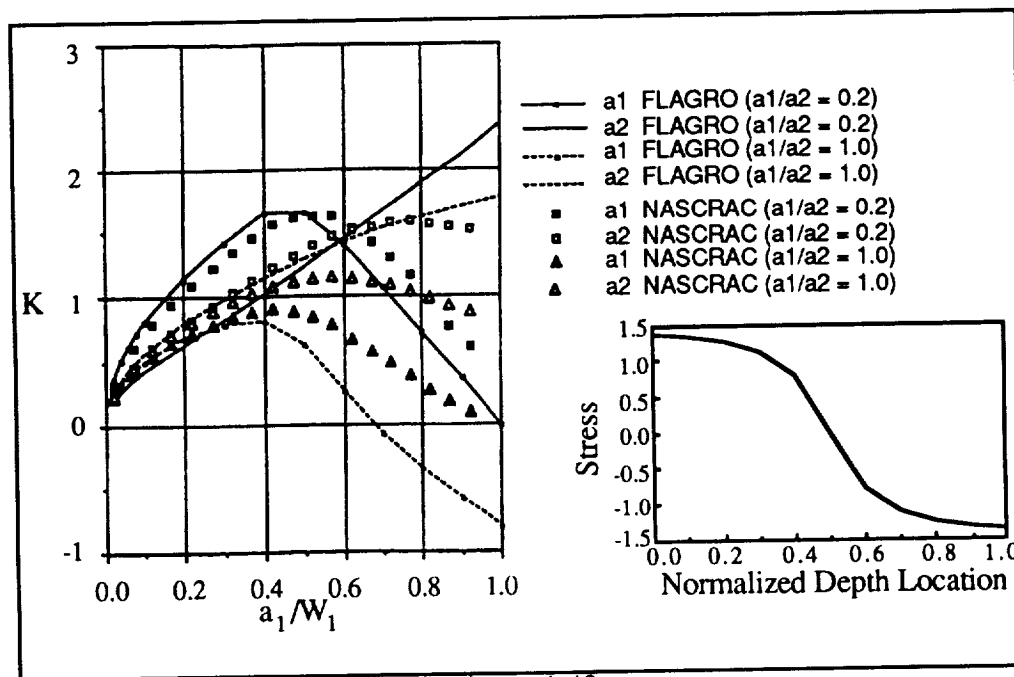


Figure 4-43
Configuration 702 in non-linear bending
Results from a round-robin study conducted by NASA/MSFC

Configuration 704 (Semi-Elliptical (Axial) Surface Crack in a Cylinder)

Figure 4-44 shows the geometry for configuration 704, Semi-Elliptical (axial) Surface Crack in a Cylinder. This configuration was verified and validated using literature sources. The NASCRAC model is reasonable for the crack tip extending into the cylinder thickness (crack tip a_1). For this crack tip, differences between NASCRAC and the references varied

from $< 10\%$ for uniform tension (Figure 4-45) and internal pressure (Figure 4-46) induced stresses to $< 20\%$ for linearly varying stresses (Figure 4-47) to $< 30\%$ for quadratically varying stresses (Figure 4-48). For cases where differences exceeded these limits ($a_1/t = 0.8$, i.e., a crack 80% through the cylinder wall thickness) NASCRAC appeared more reasonable than the references because it was more sensitive to the free surface ahead of a_1 . One drawback to K at a_1 for 704 is that NASCRAC was generally non-conservative compared to the references (see Figures 4-45 through 4-48). For K at a_2 NASCRAC predicted significantly conservative values for the cases of linearly and quadratically varying stresses with differences between NASCRAC and the references exceeding 80% for certain geometries. For uniform stresses and internal pressure loadings, NASCRAC was reasonable (differences $< 20\%$) compared to the references. The internal pressure case was not too different from a uniform stress case since the ratio of inner radius to wall thickness was 10 and the stresses in the wall varied from 10.52 psi at the inner radius to 9.52 at the outer radius. Also the discrepancies in the NASCRAC 704 model may cancel during fatigue crack growth analysis. Since the NASCRAC predictions of a_2 may be high, the crack will tend to grow along the inner surface of the cylinder more rapidly. According to reference 3, as cracks become elongated (less circular) the K value at a_1 increases. For example, given a circular crack and an elliptical crack with the same depth a_1 , the circular crack will have a lesser K at a_1 . Thus, since the NASCRAC model will be more elongated than in reality due to a conservative prediction for K at a_2 , NASCRAC will predict a correspondingly higher K at a_1 and therefore account for the underprediction of K at a_1 inherent in the NASCRAC model.

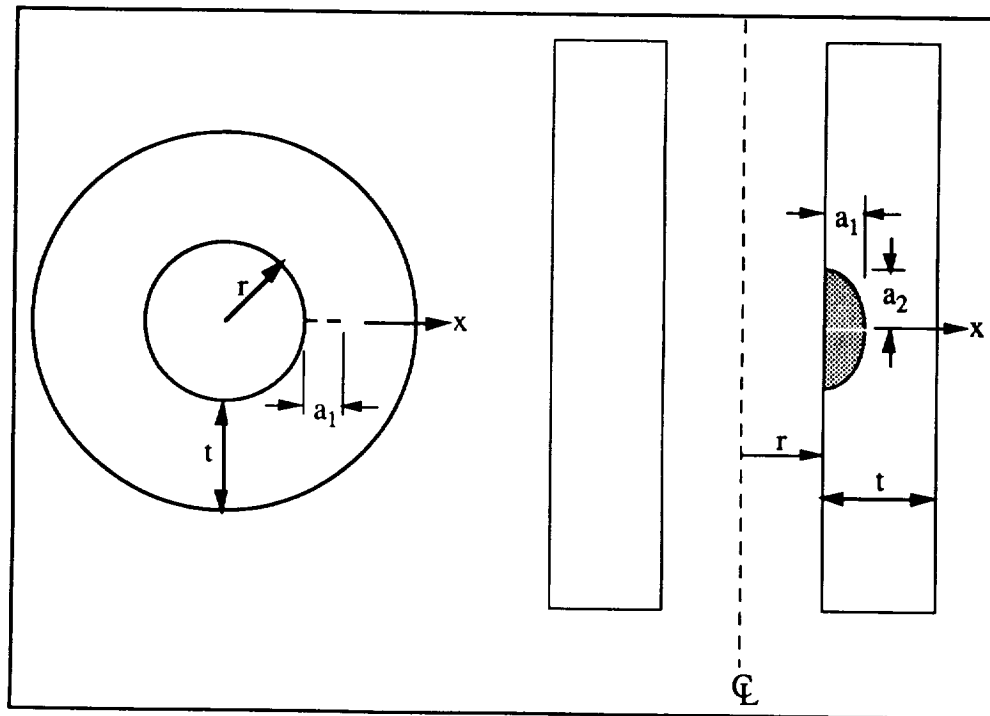


Figure 4-44
Geometry for Configuration 704

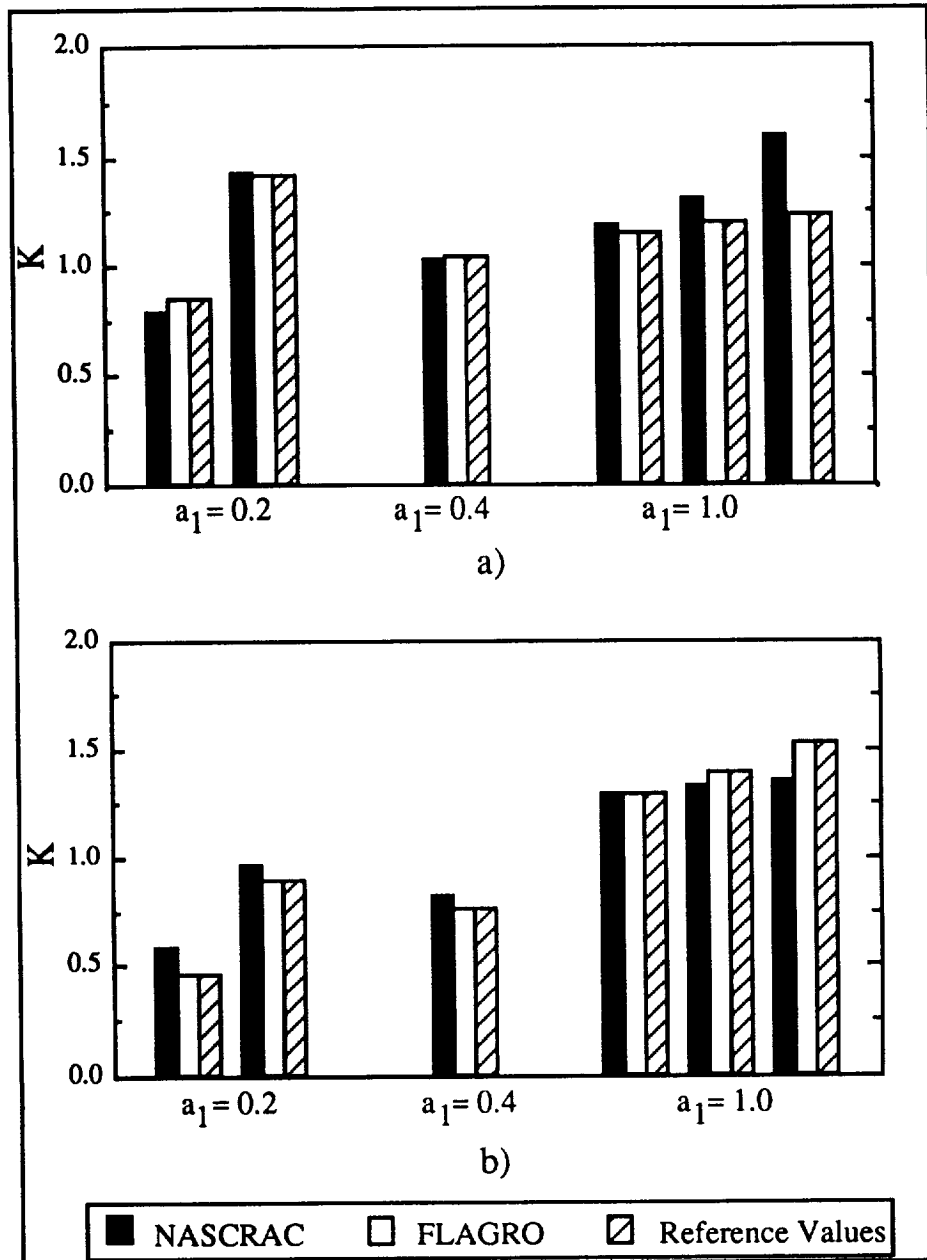


Figure 4-45
 NASCRAC, FLAGRO and Reference Values for Configuration 704
 Uniform Stress = 1 psi, $t/R_i = 0.10$
 a) K at crack tip into plate, b) K at crack tip on surface

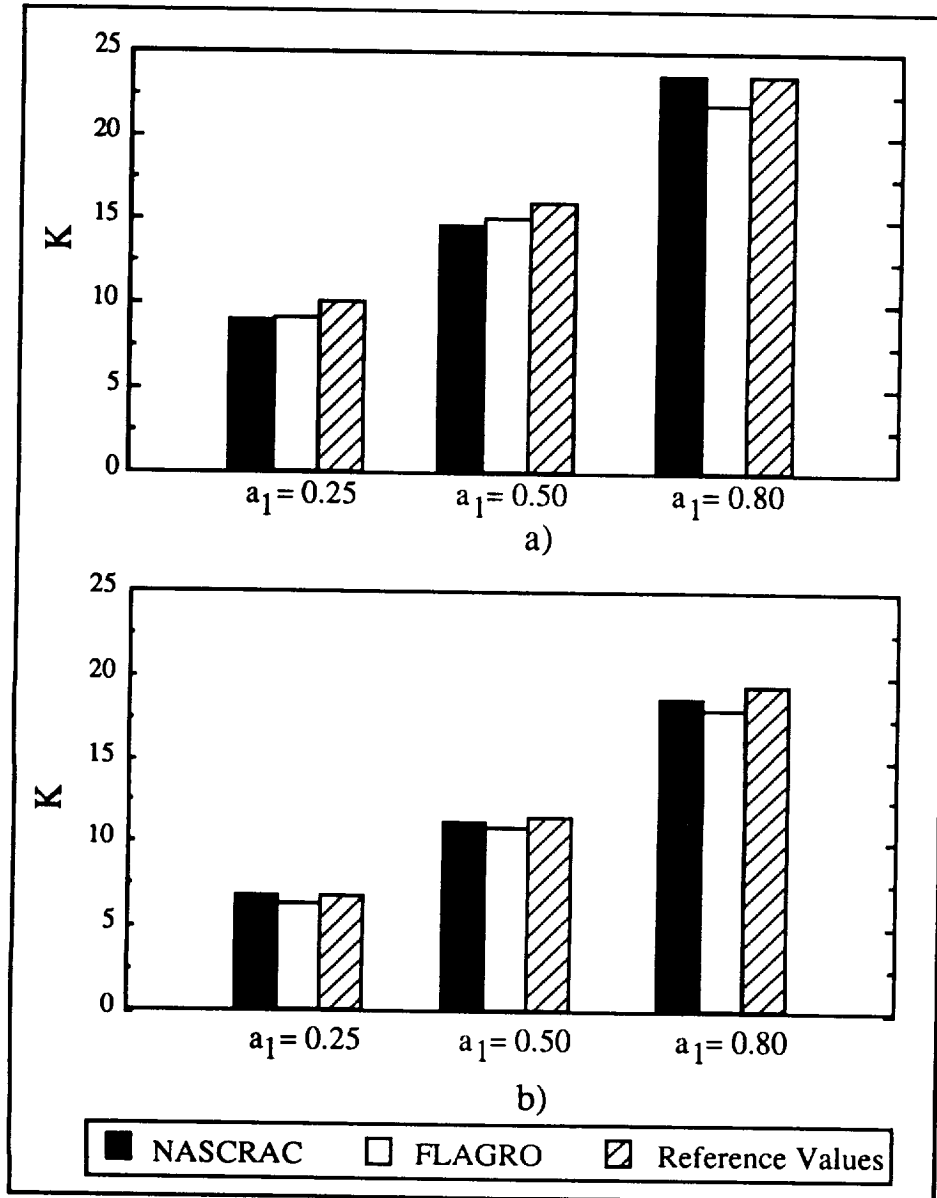


Figure 4-46
 NASCRAC, FLAGRO and Reference Values for Configuration 704
 1 psi Internal Pressure, $t/R_i = 0.10$
 a) K at crack tip into plate, b) K at crack tip on surface

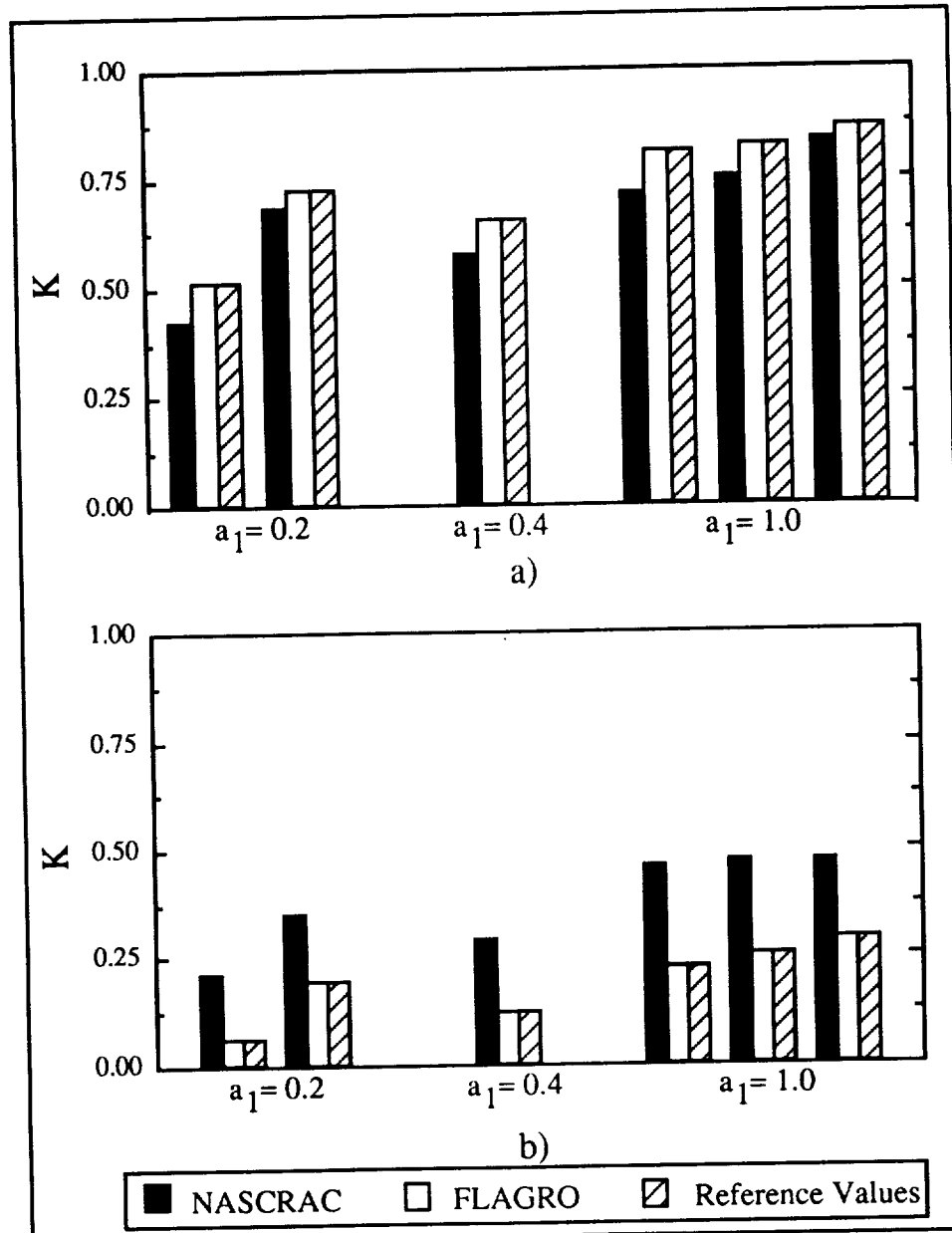


Figure 4-47
 NASCRAC, FLAGRO and Reference Values for Configuration 704
 Linear Stress = 0 psi at Crack Mouth, 1 psi at tip; $t/R_i = 0.10$
 a) K at crack tip into plate, b) K at crack tip on surface

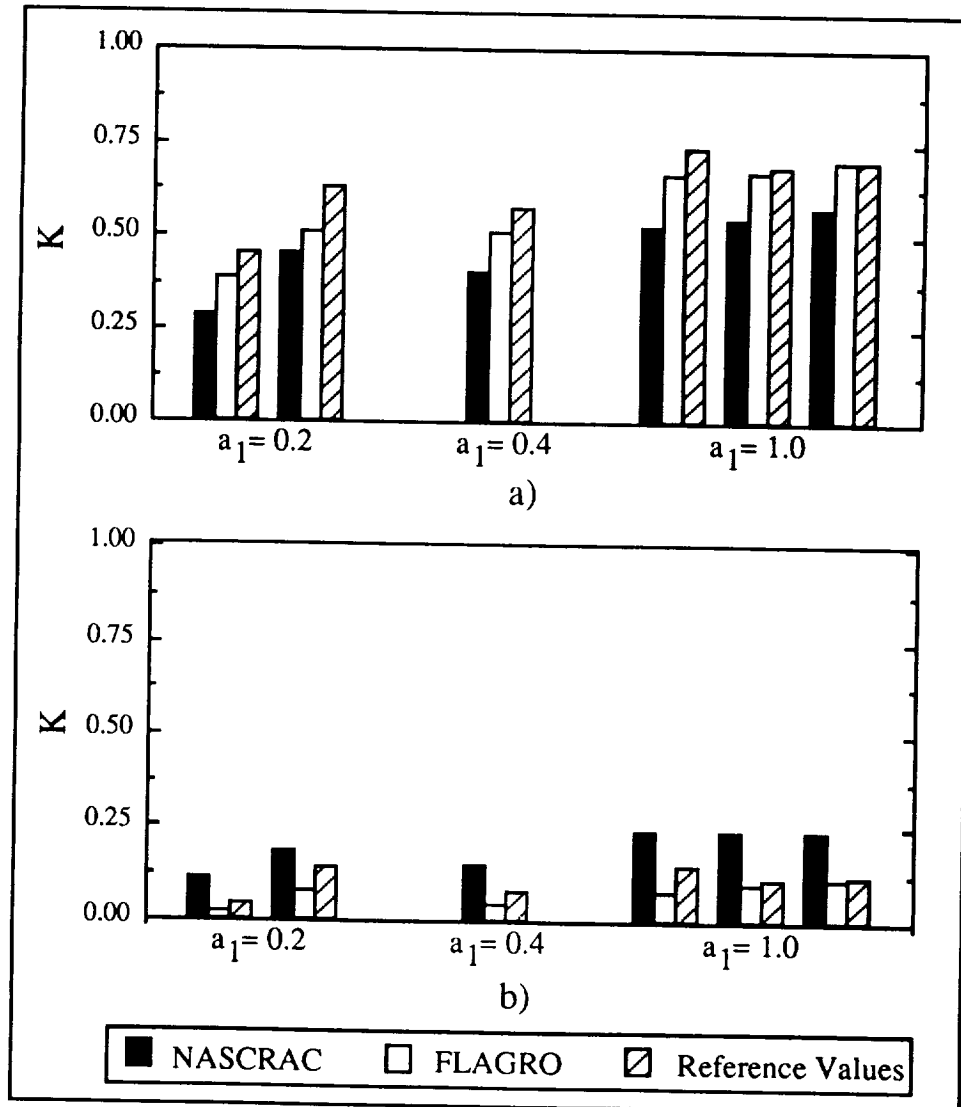


Figure 4-48
 NASCRAC, FLAGRO and Reference Values for Configuration 704
 Quadratic Stress = 0 psi at Crack Mouth, 1 psi at tip; $t/R_i = 0.10$
 a) K at crack tip into plate, b) K at crack tip on surface

800 Series Results

Configuration 801 (User-Defined K vs a Table)

Table 4-5 shows three, user defined, cases used to verify K vs a for configuration 801, User Defined K vs a Table. In the table, the second column displays the K vs a tabular values that were input into NASCRAC. The three cases in the table include a backward linear extrapolation, a linear interpolation, and a forward linear extrapolation.

Table 4-5
 Results from Configuration 801

a	K User-defined	K NASCRAC	K Calculated
Backward Linear Extrapolation			
2.0	3.7120		
2.2		3.8075	3.8075
2.4	3.9030		
Linear Interpolation			
0.1		2.9300	2.9300
1.0	3.3071		
1.2	3.3909		
Forward Linear Extrapolation			
3.8	4.9846		
4.0	5.1786		
4.2		5.4375	5.3726

The data listed in the third and fourth columns of Table 4-5 indicate an error in the forward extrapolation case. NASCRAC does not extrapolate forward correctly because a DO loop counter is used incorrectly to index the user defined data table (see code listing in Figure 4-49). If the crack length (XA) exceeds all tabulated crack lengths, the DO loop (DO 40) increments its index one final time such that ISTR = MAXDAT + 1. This sets up the interpolation indices such that K = MAXDAT + 1 and J = MAXDAT. Since there is no data for crack length (MAXDAT+1) and K (MAXDAT+1), the linear extrapolation is no longer valid and the NASCRAC computed K is simply a ratio of the final K value in the table. To correct this error, an IF/THEN construction should be used to set K=MAXDAT if the crack length (XA) exceeds tabulated values.

```

SUBROUTINE K801
C-----C
C-----C
C PURPOSE : CALCULATES K FOR 801 ; A1-K1 TABLE PROVIDED BY THE USER  C
C
C   XA=ANOW(1)
C
C INTERPOLATE TO GET K
C
30  DO 40 ISTR=2,MAXDAT
    IF(XA.LT.CRDPTH(ISTR)) GOTO 50
40  CONTINUE
    K=MAXDAT
50  K=ISTR
    J=K-1
    Y=(XA-CRDPTH(J))*XKOSIG(K)-(XA-CRDPTH(K))*XKOSIG(J)
    Y=Y/(CRDPTH(K)-CRDPTH(J))
    SIGZ=EQPARS(ITRANS,IDEF,1)
    XK(IDEF,1)=Y * SIGZ
    RETURN
END

```

Figure 4-49

Source Code for Subroutine K801 Showing Error in DO Loop Assignment

K vs a -- VARIABLE THICKNESS

NASCRAC includes *variable thickness K vs a* capabilities for seven different configurations in the 200 series (201-207). All of the configurations are through cracks and can be analyzed in FRANC using the variable thickness option. The FRANC results suggest that NASCRAC contains an inconsistency in its required input. For configurations 203, 205, 206, and 207, NASCRAC expects stresses on the crack plane to be input. This requirement is consistent with weight function theory. However, for configurations 201, 202, and 204, NASCRAC expects crack plane loads/unit plate width. The inconsistency is probably due to the weight functions coded in NASCRAC. For configurations 203, 205, 206, and 207, NASCRAC uses a generic weight function routine (FUNCTION GENRIF). The function coefficients for this routine were generated offline for each relevant crack configuration and hardwired into NASCRAC. For configurations 201, 202, and 204, NASCRAC uses weight functions obtained from literature sources (FUNCTION FCT201, FUNCTION FCT202, FUNCTION FCT204). In the literature, these functions are presented in terms of load/unit width (P/B). NASCRAC employs FCT201, FCT202, and FCT204 exactly as in the literature and thus a P/B input is necessary. The problem can be illustrated with the source code listed in Figure 4-50. This figure lists a skeleton of the typical NASCRAC subroutines used to compute K solutions. The first routine, Kxxx, calls a Gaussian quadrature integration routine, QINTxx, using an external function, FCTxxx, as a calling parameter. The x's represent the appropriate configuration number (e.g., 201). The external function FCTxxx consists of the weight function for the Kxxx configuration. Note that at the bottom of the FCTxxx function, the function is multiplied by the thickness (TX) before returning to Kxxx. After returning to Kxxx, the thickness (THICKX) is divided out; thus, the thickness operations have zero net effect in terms of units. Since the thickness operations produce no effect in terms of units, the resulting K value is dependent on the units of the inputs. Thus, for the weight functions taken from the literature, if stresses are input into NASCRAC instead of load/unit width, the calculated NASCRAC results will be in error by a factor of thickness.

```

SUBROUTINE Kxxx
C
C   EXTERNAL FCTxxx
C
C   INTEGRATION USING THE INFLUENCE FUNCTION
C
C   CALL QINTxx(FCTxxx,Y)
C
  IF (VTHICK EQ 1) THEN
    NK(DEF,1)=Y/THICKX(A)
  ELSE
    NK(DEF,1)=Y
  ENDIF
RETURN
END
*****
FUNCTION FCTxxx(XOA)
.
.
CALL STRINT(X,Y,Z,SIGMA)
FCTxxx= SIGMA * F * A
  IF (VTHICK EQ 1) THEN
    TX=THICKX(A)
  ELSE
    TX=1.
  ENDIF
FCTxxx = FCTxxx*TX
RETURN
END

```

Figure 4-50
Variable Thickness Operations in K Solution Subroutines

Table 4-6 illustrates the error described above. In this table three sets of results are listed: FRANC results with a variably thick model, NASCRAC results using stress inputs, and NASCRAC results using load/unit width inputs. Table 4-6 lists results for both configuration 202, which uses a weight function from the literature, and configuration 203, which uses a generic weight function generated for NASCRAC. The results clearly show the inconsistency in the expected inputs for NASCRAC's variably thick K solutions. For configuration 202, NASCRAC agrees with FRANC when load/unit width values are input to NASCRAC; conversely, for configuration 203, NASCRAC agrees with FRANC when stress values are input. For both configurations, when the variable thickness option is employed but the thickness is uniform with a value of unity (Case $t = 1.0$ in Table 4-6) the NASCRAC results for both stress and load/unit width inputs are identical and agree with FRANC results. Case 2, which was also computed with the variable thickness option and a thickness table with constant values not equal to unity, shows the described inconsistency. The inconsistency also occurs in the constant (uniform) thickness K solutions but is not evident in the results since the crack plane stresses and the crack plane loads/unit width are identical for a uniform thickness ($= 1$). This result is evident in the subroutines listed above in Figure 4-50 where the thickness values TX and THICKX(A) are set to unity (see the highlighted IF-THEN statement) for the constant thickness option. To correct the inconsistency the required input units for each configuration should be explicitly stated in

the documentation and displayed by the user interface. A more rigid resolution of the inconsistency is to recode NASCRAC to expect stress values on the crack plane in all situations.

Table 4-6
K Results for Variable Thickness Option

Case	a	Configuration 202			a	Configuration 203		
		NASCRAC (stresses)	NASCRAC (load/width)	FRANC		NASCRAC (stresses)	NASCRAC (load/width)	FRANC
t = 1.0	0.5	0.134	0.134	0.134	1.0	0.206	0.206	0.201
	1.5	0.241	0.241	0.240	3.0	0.495	0.495	0.495
	2.5	0.344	0.344	0.342	5.0	1.117	1.117	1.110
t = 2.5	0.5	0.021	0.053	0.054	1.0	0.083	0.207	0.080
	1.5	0.039	0.097	0.096	3.0	0.199	0.498	0.198
	2.5	0.055	0.138	0.138	5.0	0.448	1.120	0.443
t = 0.5 + 0.2x	0.5	0.062	0.096	0.099	1.0	0.227	0.141	0.210
	1.5	0.092	0.154	0.164	3.0	0.414	0.354	0.371
	2.5	0.109	0.192	0.218	5.0	0.730	0.748	0.652
t = 2.5 - 0.2x	0.5	0.076	0.109	0.109	1.0	0.068	0.160	0.069
	1.5	0.173	0.226	0.220	3.0	0.239	0.510	0.271
	2.5	0.316	0.376	0.366	5.0	0.729	1.431	0.863
t = 1.0 + 0.2x + 0.02x ²	0.5	0.127	0.135	0.139	1.0	0.368	0.189	0.341
	1.5	0.172	0.205	0.221	3.0	0.684	0.392	0.577
	2.5	0.184	0.241	0.277	5.0	1.090	0.733	0.897

To summarize the verification and validation effort for NASCRAC's variable thickness option, the NASCRAC variable thickness capability is valid for uniform thicknesses $\neq 1.0$ and linearly varying thicknesses for all variably thick configurations (201-207) as partially evidenced by Table 4-6 if the correct loading (as discussed above) is applied. Table 4-6 also suggests that NASCRAC is valid for small cracks and quadratically varying thicknesses. The discrepancies between NASCRAC and FRANC for quadratically varying thicknesses and larger cracks may, in part, be due to the coarseness of the discretization applied in NASCRAC for the stress and thickness tabular inputs.

J vs a

NASCRAC contains J vs a capabilities for eight configurations. In general, the J vs a capabilities are valid; however, several exceptions were determined. Configuration 303, a circumferential through crack in a cylinder, is the most significant exception because it includes a runtime error. The remaining exceptions hinge on the h_1 table included in NASCRAC and the method of calculating J_e , the elastic J integral. h_1 is a dimensionless function included in the J_p formulation. It is dependent on a/b , the crack length to specimen width ratio, and n , a hardening exponent for the Ramberg-Osgood constitutive relationship. These relationships are expressed in the following equations:

$$J_p = a \sigma_y \epsilon_y c \frac{a}{b} h_1 \left(\frac{P}{P_0} \right)^{n+1}$$

$$h_1 = f\left(\frac{a}{b}, n\right)$$

$$\frac{\epsilon}{\epsilon_0} = a \left(\frac{\sigma}{\sigma_0} \right)^n$$

The worst h_1 table errors occur in the plane strain case of configuration 203. A limited comparison between NASCRAC and the reference table is given in Table 4-7. A majority of the NASCRAC entries in this table differ from the reference table values; therefore, an erratum sheet should be released for the current version of NASCRAC identifying the errors. This erratum sheet would allow analysts to correct their current calculations offline. In addition the configuration 203 h_1 table for plane strain should be corrected prior to releasing future NASCRAC versions. Several less significant h_1 errors were also discovered for configurations 101, 202, and 204, and should be corrected in future versions of NASCRAC.

Table 4-7
NASCRAC and Reference h_1 Values for Configuration 203 in Plane Strain

a/b	n = 1		n = 3		n = 5		n = 10		n = 13		n = 16	
	NAS	ref	NAS	ref	NAS	ref	NAS	ref	NAS	ref	NAS	ref
1/8	4.95	5.01	8.57	9.09	11.5	12.7	16.1	21.7	18.1	27.3	19.9	34.4
1/4	4.34	4.42	4.64	5.16	3.82	4.50	2.17	2.74	1.55	1.93	1.11	1.82
3/8	3.88	3.97	2.63	2.88	1.68	1.92	0.54	0.70	0.28	0.40	0.14	0.22
1/2	3.40	3.45	1.69	2.02	0.93	1.22	0.21	0.38	0.09	0.19	0.04	0.10
5/8	2.86	2.89	1.30	1.70	0.70	1.11	0.15	0.42	0.06	0.24	0.03	0.14
3/4	2.34	2.38	1.25	1.56	0.77	1.13	0.23	0.59	0.12	0.41	0.06	0.29
7/8	1.91	1.93	1.37	1.43	1.10	1.18	0.70	0.81	na	na	na	na

For J_e , the discrepancies between the NASCRAC computed value (computed using the elastic version of the coded J_p formulation and an effective crack length) and a J_e computed from K , E , and an effective crack length were observed to be more severe as the analysis transitioned into the elastic-plastic and plastic regime. Although some of these discrepancies were significant (differences of 50-60%), the contribution of J_e towards the total J for these cases was insignificant; therefore, NRC does not recommend any immediate action on this discrepancy. NRC does recommend that NASA address the discrepancy in the future by resolving the issue of effective crack length for the formulation $J_e = K^2/E$.

As noted above, a runtime error was discovered in configuration 303. A second error, the definition of the mean radius of the cylinder, was also found in this capability. The runtime error, a *divide by zero* error, occurred because the variable PI (Figure 4-51) was not defined in subroutine *GETJS* and therefore was automatically set to zero by the VAX. The mean radius was incorrectly defined in *GETJS* as the inner radius plus one-half of the arc length (*WIDTHS(1)*), not the inner radius plus one-half the cylinder wall thickness (*WIDTHS(2)*). Both errors were corrected offline. Results from the corrections, which are given in table 4-8, are in good agreement with reference 5. In table 4-8, the *Reference* column is the benchmark for comparison, the *PI NASCRAC* column contains results from a corrected version in which only the first error, the assignment of PI , was corrected, and, finally, the *PI and Rm NASCRAC* column contains results from a version in which both errors were corrected. The results in the table clearly indicate that simply defining PI did not result in a valid solution.

```

SUBROUTINE GETJS (XFCTR)
.
.
XNC=SHARDN
.
C
C   THRU WALL CRACK IN A CYLINDER
C
CAL=0.0625
CAH=0.5
XNL=1.
XNH=7.
AB=ANW(1)/(PI*(WIDTHS(3)+0.5*WIDTHS(1)))
CALL WARNJ(AB,CAL,CAH,XNC,XNL,XNH)
B=WIDTHS(1)
T=WIDTHS(2)
RIOB=(WIDTHS(3)+0.5*WIDTHS(1))/WIDTHS(2)
IF (RIOB.LE.7.5) THEN
  CNAME='TCT5'
ELSE IF (RIOB.GT.7.5 .AND. RIOB.LE.15.) THEN
  CNAME='TCT1'
ELSE
  CNAME='TCT2'
END IF
CALL JINT
RETURN
END

```

Figure 4-51
Condensed Subroutine GETJS Showing Errors in *PI* and *mean radius* Assignments

Table 4-8
Results from a Corrected Version of NASCRAC Configuration 303 J vs a Capability

Case	J_e Reference	J_e Pi NASCRAC	J_e Pi&Rm NASCRAC	J_p Reference	J_p Pi NASCRAC	J_p Pi&Rm NASCRAC	J_{total} Reference	J_{total} Pi NASCRAC	J_{total} Pi&Rm NASCRAC
1	2.20E+0	2.14E+0	2.29E+0	1.87E+2	1.61E+2	1.88E+2	1.90E+2	1.63E+2	1.90E+2
2	5.48E+0	5.15E+0	5.88E+0	7.07E+2	4.91E+2	7.08E+2	7.13E+2	4.96E+2	7.14E+2
3	2.43E+1	1.59E+1	2.54E+1	6.63E+3	2.41E+3	6.62E+3	6.65E+3	2.42E+3	6.65E+3
4	3.64E+2	9.95E+1	6.17E+3	2.91E+6	3.60E+4	2.53E+6	2.91E+6	3.61E+4	2.54E+6
5	2.34E-3	2.26E-3	2.41E-3	2.57E-7	2.20E-7	2.57E-7	2.34E-3	2.26E-3	2.41E-3
6	5.62E-3	5.38E-3	6.06E-3	9.70E-7	6.74E-7	9.71E-7	5.62E-3	5.38E-3	6.06E-3
7	1.99E-2	1.57E-2	2.26E-2	9.09E-6	3.30E-6	9.08E-6	1.99E-2	1.57E-2	2.26E-2
8	1.80E-1	7.34E-2	2.34E-1	3.99E-3	4.94E-5	3.47E-3	1.84E-1	7.34E-2	2.38E-1
9	2.34E-3	2.26E-3	2.41E-3	5.43E-1	5.08E-1	5.43E-1	5.46E-1	5.11E-1	5.45E-1
10	5.62E-3	5.38E-3	6.06E-3	1.37E+0	1.21E+0	1.36E+0	1.37E+0	1.22E+0	1.37E+0
11	1.98E-2	1.57E-2	2.26E-2	5.10E+0	3.54E+0	5.09E+0	5.11E+0	3.56E+0	5.11E+0
12	1.80E-1	7.33E-2	2.33E-1	5.54E+1	1.65E+1	5.24E+1	5.56E+1	1.66E+1	5.26E+1

COA vs a

Table 4-9 lists errors and corrections for the COA capabilities in NASCRAC. Each error is described in detail in sections following the table.

Table 4-9
Errors in NASCRAC's COA Capabilities

Configuration	Error	Correction
201	Plane strain assumption*	Document the assumption
202	Plane strain assumption* H/W ≥ 2 assumption*	Document the assumptions
302	Typographical error in source code	Correct spelling in source
303	Typographical error in source code	Correct spelling in source

* Not an error per se but an undocumented assumption that could lead to misinterpretations

Configuration 201 (Crack in an Infinite Plate)

For configuration 201, NASCRAC calculates the area for the case of plane strain but does not identify this assumption to the user. Although the coded solution is not in error, if the solution is used to calculate COA for a case of plane stress, the computed results would underestimate the closed-form plane stress solution by approximately 11% for aluminum, where Poisson's ratio is high (0.33).

Configuration 202 (Center Cracked Panel)

For configuration 202, as in configuration 201, NASCRAC uses the plane strain assumption but does not document it. If the coded solution were used to calculate COA for a case of plane stress, the results would be non-conservative by approximately 11% for an aluminum component where the Poisson's ratio is high (0.33). Additionally, the coded solution for configuration 202 is only valid for a plate height to width ratio $H/W \geq 2$. Warnings should be included in the output when this criteria is not met.

Configuration 302 (Through Crack in a Cylinder - Axial)

In configuration 302 a typographical error occurs in the source code which leads to COA calculations out of the range of the implemented solution. NASCRAC assigns a variable ALP but misspells the name in an IF-THEN statement several lines later. This error is highlighted in the *GETCOA* source code listed in Figure 4-52 (See the first two highlights in the figure.) This error causes crack opening areas to be calculated for $ALP > 5$ even though the reference solution is only valid for $a \leq 5$. This error can easily be fixed in future NASCRAC releases by implementing the correctly spelled variable.

Configuration 303 (Through Crack in a Cylinder - Circumferential)

A typographical error in configuration 303 causes NASCRAC to compute COA results that are non-conservative by 30-40% in some cases. The error is related to the variable ALPH. During calculation of GOALPH, ALPH is misspelled as shown in the final two highlighted lines of *GETCOA* source code listed in Figure 4-52. The misspelling causes the aforementioned 30-40% non-conservative results when $1 < \text{ALPH} \leq 5$. This error can easily be fixed in a future NASCRAC release by implementing the correctly spelled variable.

```

SUBROUTINE GETCOA
.
302 CONTINUE
.
R=WIDTHS(3)+WIDTHS(2)/2.
ALP=ANOW(1)/SQRT(WIDTHS(2)*R)
IF (ALP.GT.0.0 .AND. ALP.LE.1.0) THEN
    GOALP=ALP*ALP+0.625*ALP**4
ELSE IF (ALP.GT.1. .AND. APL.LE.5) THEN
    GOALP=.14+0.36*ALP*ALP+0.72*ALP**3+0.405*ALP**4
ELSE
    WRITE(NFLLPT,2001)
2001 FORMAT(1X,'ALPHA MUST BE BETWEEN 0 AND 5')
    RETURN
END IF
XK(IDEF,1)=SIGINF*2.*3.14159*WIDTHS(2)*R*GOALP/YOUNGS
&          *(1.-POISSN*POISSN)
GOTO 998
C
303 CONTINUE
.
R=WIDTHS(3)+WIDTHS(2)/2.
ALPH=ANOW(1)/(SQRT(R*WIDTHS(2)))
IF (0.0 .LT. ALPH .AND. ALPH.LE.1) THEN
    GOALPH=ALPH**2+0.16*ALPH**4
ELSE IF (1. .LE. ALPH .AND. ALPH.LE. 5.0) THEN
    GOALPH=0.02+0.81*ALPH**2+0.30*APLH**3+0.03*ALPH**4
ELSE
    WRITE(NFLLPT,2001)
    GOTO 998
END IF
XK(IDEF,1)=SIGINT*2.*3.14159*R*WIDTHS(2)*GOALPH/YOUNGS
          *(1.-POISSN*POISSN)
GOTO 998
.
RETURN
END

```

Figure 4-52
Typographical Errors in *GETCOA* for Configurations 302 and 303

SUMMARY and RECOMMENDATIONS

Verification and validation of NASCRAC's K vs a , J vs a and COA vs a capabilities have been completed. Several limitations of NASCRAC and several minor errors such as typographical mistakes were found. Additionally a few significant discrepancies in NASCRAC results were observed and documented. The K vs a capabilities for the 100 series (standard specimens), 200 series (through thickness cracks in planar bodies), 300 series (through thickness cracks in shells), 400 series (one degree-of-freedom cracks in bodies of revolution), and 500 series (buried cracks) cracks generally compared well with the reference solutions. The 600 series (corner cracks) and 700 series (surface cracks) cracks tended to differ more from the reference solutions. Variable thickness K vs a solutions were in good agreement with the references when consistent input quantities were included. J vs a solutions showed good agreement with the references except for cases where h_1 table entries were incorrect in NASCRAC. Additionally J_e , the elastic component of J , diverged significantly from reference results after the solution had entered the plastic zone. This result is expected and does not affect the validity of the total J solution because the elastic component is insignificant in this region. Finally, other than minor typographical errors, the COA vs a solutions in NASCRAC compared favorably to reference 12, which was the source for the coded solutions.

Typographical errors that were uncovered should be corrected as soon as possible and released in an updated version. The corrections should include the errors found in K vs a for configurations 101, 601, and 602; J vs a h_1 tables and for configuration 303; and COA vs a for configurations 302 and 303. A correction to the screen message for configuration 104 geometry (the message defines the relationship between the specimen width and length) should also be made while editing the typographical errors.

The updated version should include coded error flags which force the user to input values within the geometry domain of the coded solution. These flags are necessary for geometry definitions for configurations 205, 207, 208, 301, 302, 403, and 404.

Documentation which fully discusses the expected load inputs for the variable thickness K vs a solutions should be released as soon as possible. In a future version of NASCRAC the K vs a solutions should be recoded so that stress values will always be the required input quantity. This recoding will ensure consistency throughout the code and also consistency with weight function theory.

The K solution for configuration 404 should be reformulated in future releases of NASCRAC. The current solution assumes a geometry that is easily described with two variables. This geometry is reasonable for static K vs a analyses where a is less than the radius of the cylinder; however, during fatigue crack growth, this model would grow the crack in a non-conservative manner.

For configurations 601 and 602, NRC recommends that NASA/MSFC sponsor a parametric finite element analysis to develop an upgrade to these solutions which more accurately models the configuration. The analytical studies should include hole diameter, plate width, plate thickness, and, perhaps, pin load distribution as parameters. For configuration 605, NASA/MSFC should develop an improved solution for non-uniform load configurations.

Weight functions for 702, 704, and 705 were derived from the 703 weight function and adjusted for geometry. Application of this function to the 702 and 704 geometry is questionable due to curvature effects, especially at crack tip a_2 , which is curved in the case

of 703 and 705 but straight in the case of 702 and 704. To increase confidence, NASA should develop independent weight function solutions for 702 and 704 for incorporation into NASCRAC. It may be possible to derive such independent solutions from the work of Newman and Raju.

In *Section IV, Results*, under configuration 704, a hypothesis from reference 3 was paraphrased: As cracks become elongated (less circular) the K value at a_1 increases. For example, given a circular crack and an elliptical crack with the same depth a_1 , the circular crack will have a lesser K at a_1 . If this statement is true, then the tendency of NASCRAC to underpredict K at a_1 for configuration 704 will cause the crack to become less circular and more elongated during fatigue crack growth simulations. Since the crack will be more elongated than in reality, NASCRAC will predict a higher K value at a_1 compared to its prediction for a circular crack with the same crack depth a_1 , a crack which may more closely model the real crack. This hypothesis from reference 3 and the application of it to the NASCRAC 704 model should be verified through finite element studies and incorporated into any new algorithms developed for configuration 704.

The use of RMS averaging to calculate K values for three dimensional cracks such as configurations 601, 602, 702 and 704 is highly suspect for bending loads. NASA/MSFC should develop a consensus on this approach to calculating K before employing NASCRAC computed K 's for these configurations under bending. Results presented in this report suggest that NASCRAC should not be employed for these cases.

To conclude, several minor corrections to NASCRAC version 2.0 and its accompanying documentation can be completed with minimal effort and released in an updated version in the near term. Other more complicated questions about specific NASCRAC solutions will require analytical efforts to determine more acceptable solutions than the ones currently in NASCRAC. These efforts should be sponsored by NASA/MSFC. Finally use of NASCRAC version 2.0 as a tool for computation of K vs a , J vs a , and COA vs a should be acceptable to NASA/MSFC except as noted in this report.

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